Engineering Design and Decision Making

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Unpleasant surprises

$2B San Francisco transit center closed due to cracks

Photo: KQED and NBC

Emerging challenges

Automation in design, manufacturing, construction

Custom design, unique settings, novel material, evolving demand

Uncertain loads; climate adaptation

Pressure on cost and resource usage

optimization under uncertainty

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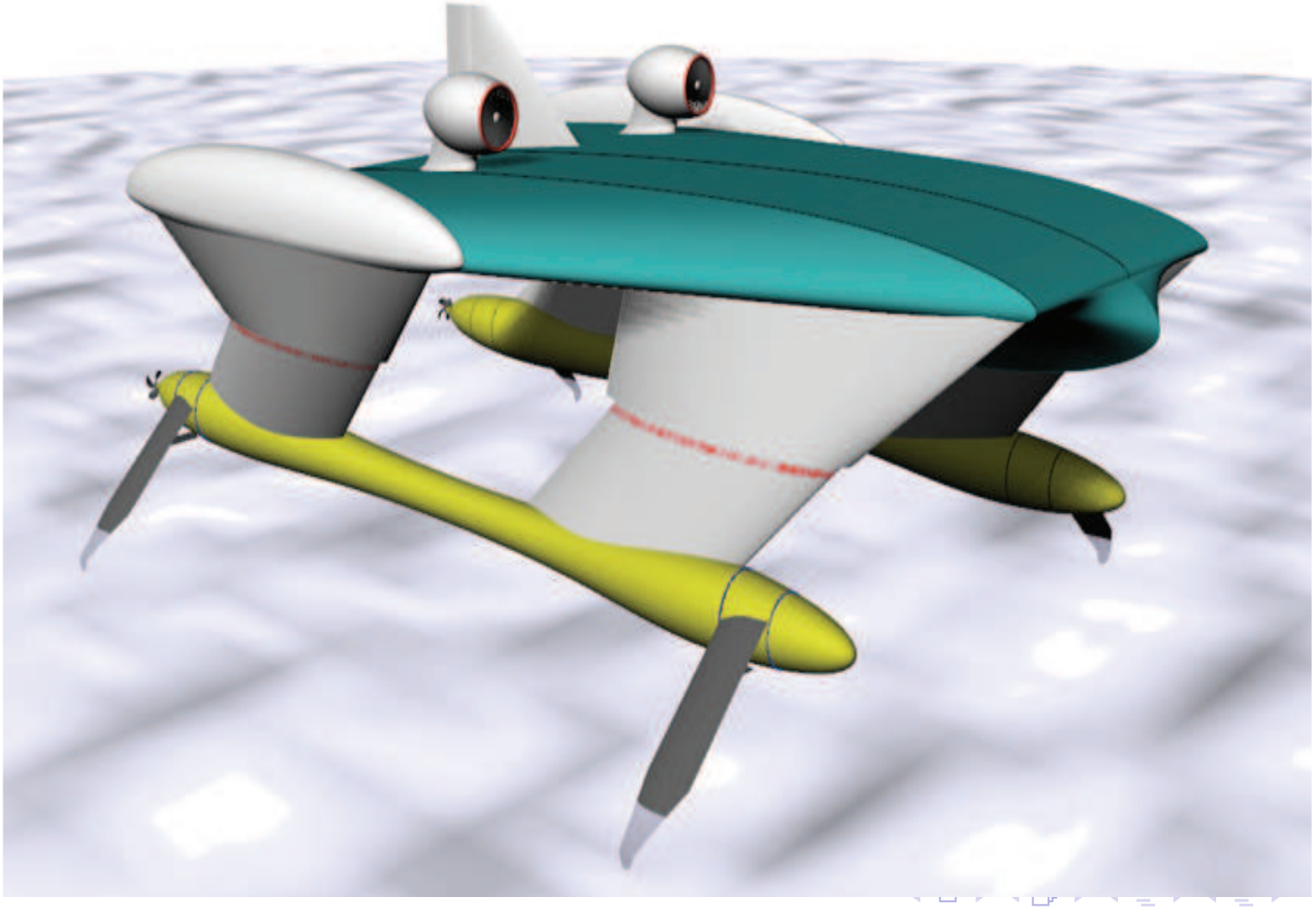
Need to leverage:

- computing power and data storage
- sensing capability
- data analytics
- predictive models of social, physical, and cyber systems

Case study: design of high-speed vessel

novel concept and material with complicated physics

multi-disciplinary



Role of probability and statistics

Early approaches: tradition, testing, safety factors

Emerging **uncertainty quantification**:
Benjamin & Cornell '70; Ang & Tang '75

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(e.g., $Y = \text{load} - \text{strength}$)

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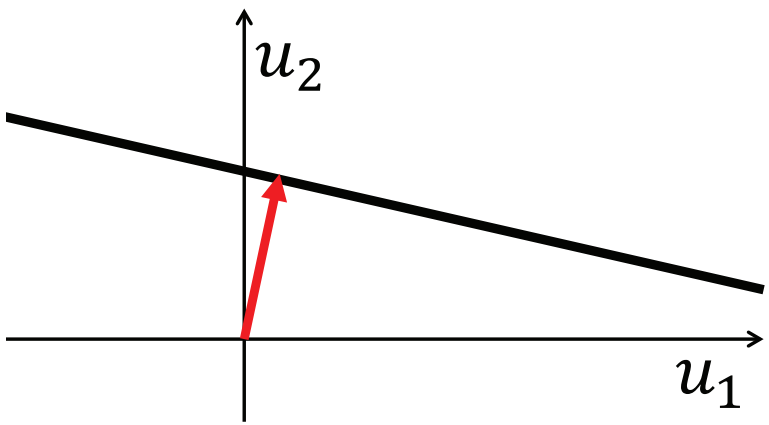
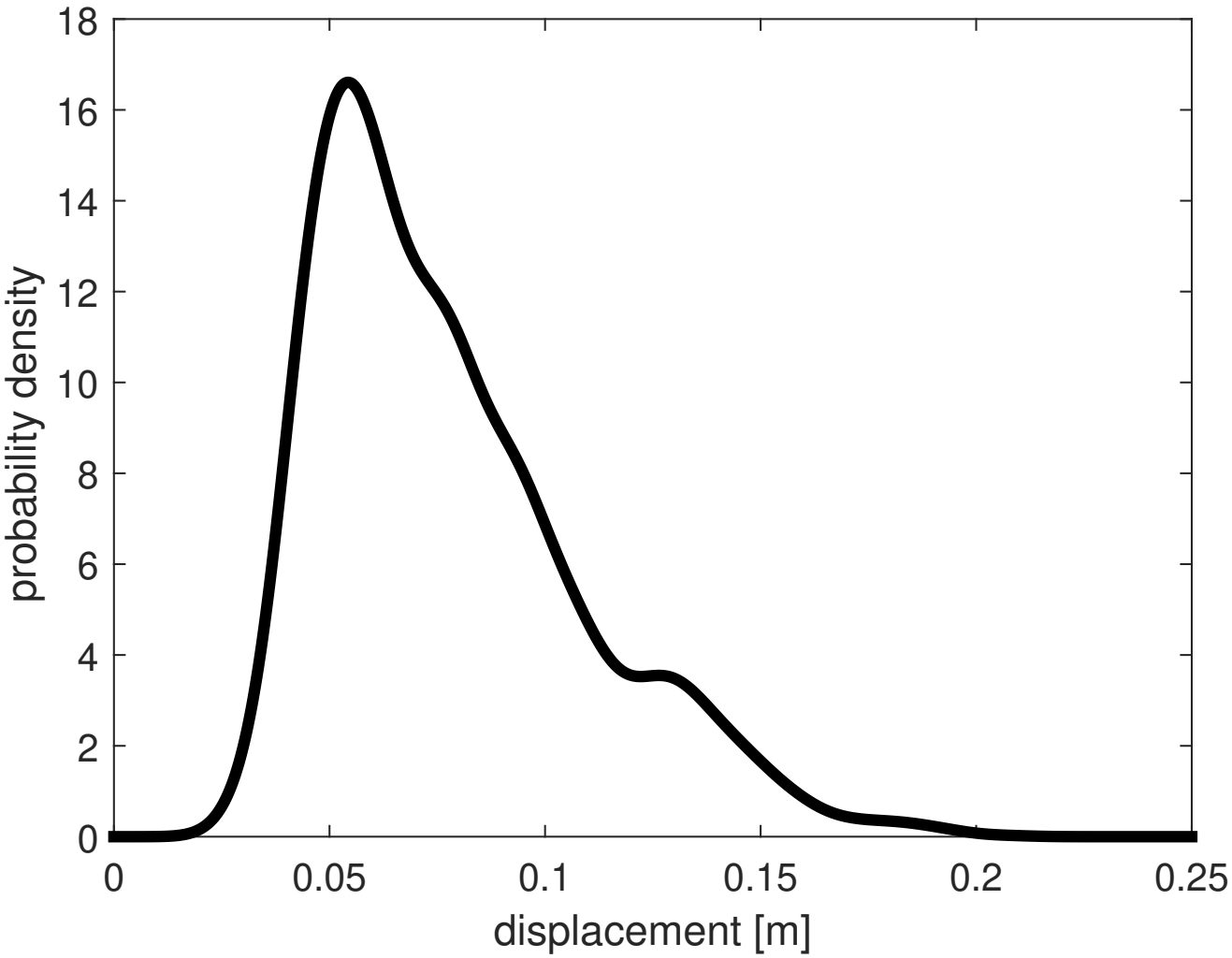
Hasofer & Lind '74: for $Y = a^\top V + b$, with normal vector V
standardize: $Y = \tilde{a}^\top U + \tilde{b}$
HL-reliability index: distance from origin to $\tilde{a}^\top u + \tilde{b} = 0$



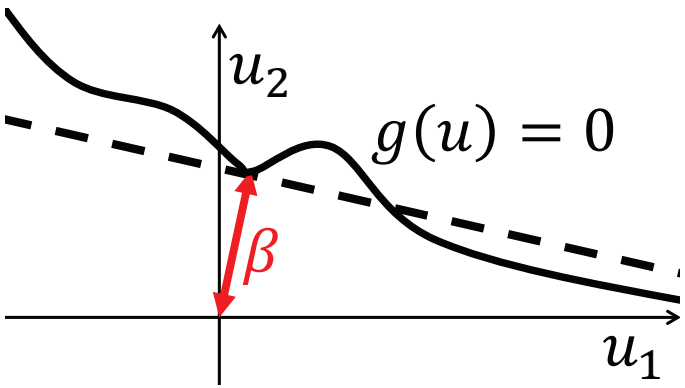
A nonnormal world . . .



Uncertain tip displacement of hydrofoil under random cavitation index and material properties

Failure probability

Rackwitz & Fiessler '78: for $Y = g(U)$, possibly nonlinear
linearization of g at nearest point to origin (FORM)
probability of failure: $\text{Prob}(Y > 0) \approx \Phi(-\beta)$



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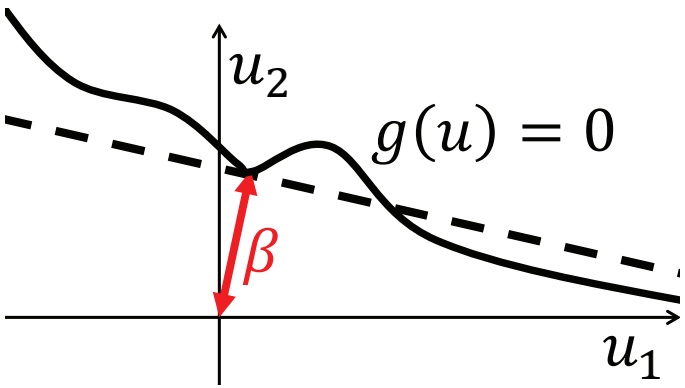


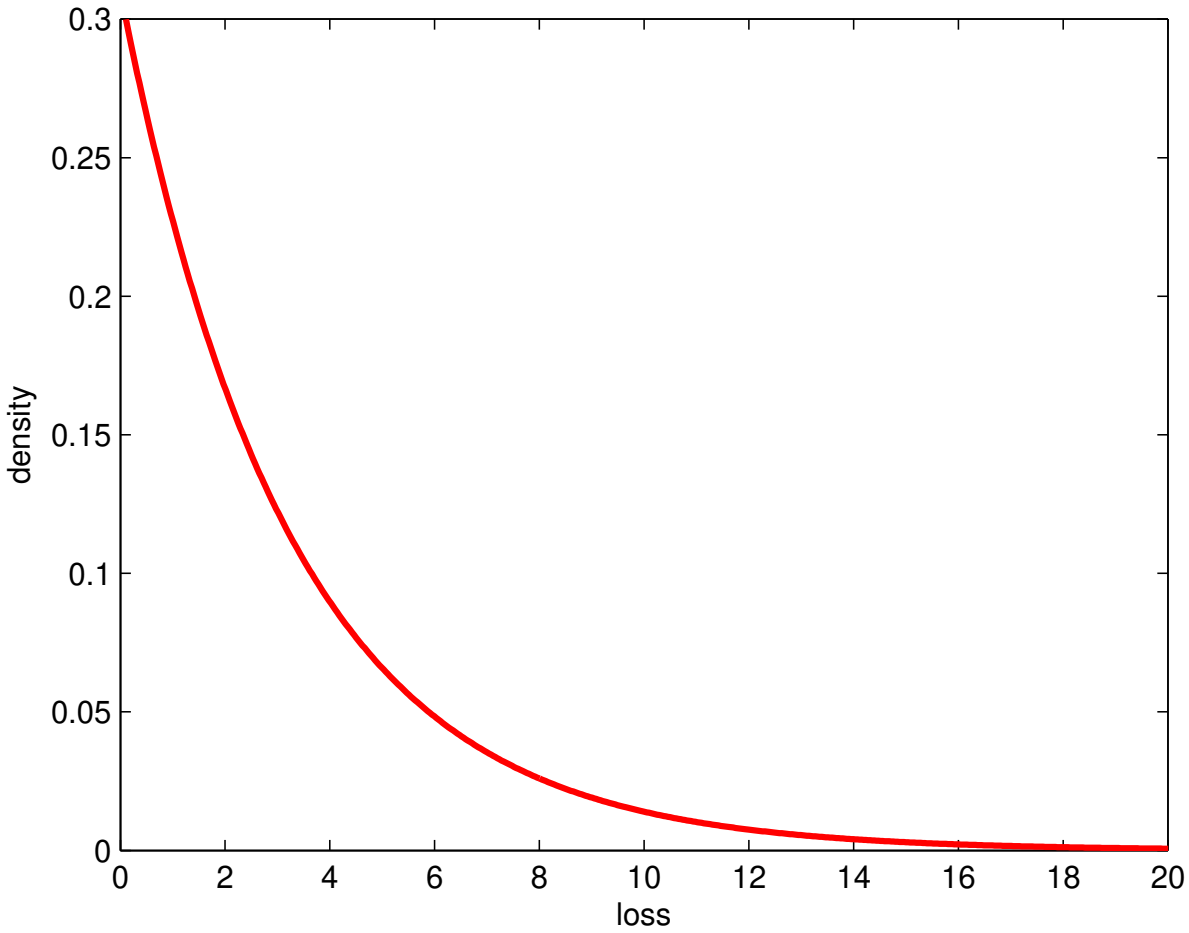
Would like to optimize the design:

- g also depends on design variables x
- FORM: bilevel optimization (lack of constraint qualification)
- Direct optimization of $\text{Prob}(g(x, V) > 0)$ difficult (nonconvex, noncontinuous)
  Uryasev '95; Royset & Polak '07; Van Ackooij & Henrion '14

More comprehensive: probability distribution

Performance-based engineering '90s: estimate the distribution

Hazard, structural, damage, loss analysis (Gunay & Mosalam '13)



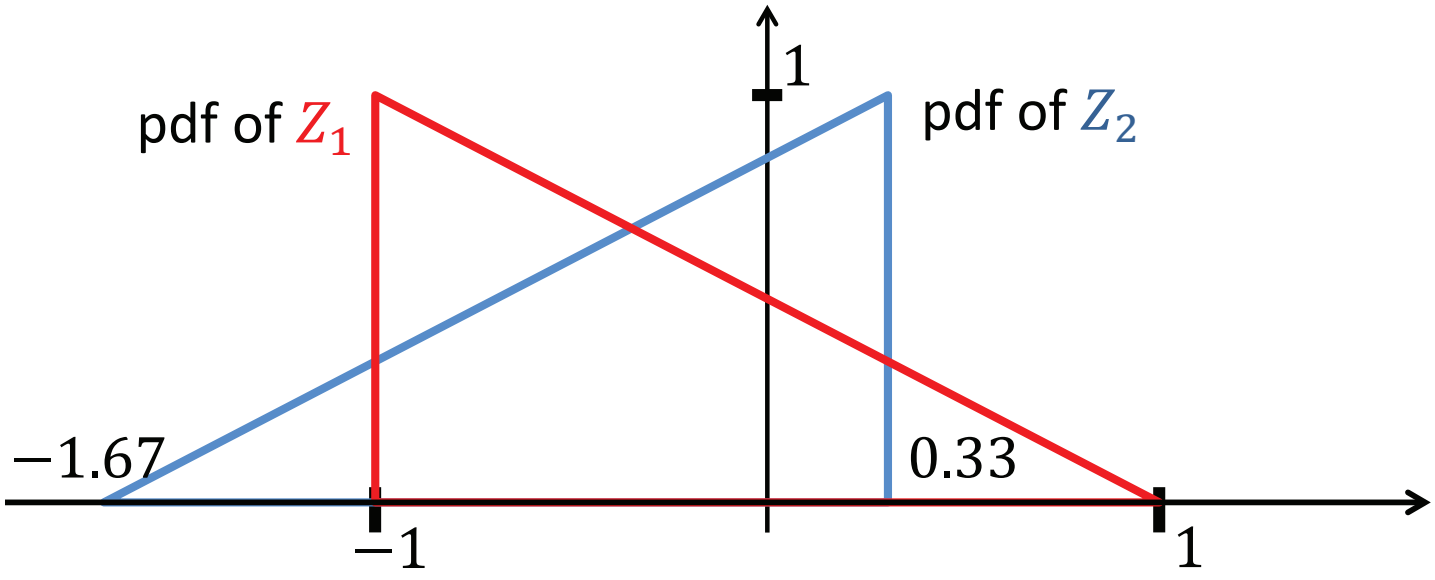
50-year loss from earthquakes in Vancouver area (billon CAD)

Mahsuli & Haukaas, 2013

How to compare probability distributions?

Design 1: uncertain response Z_1

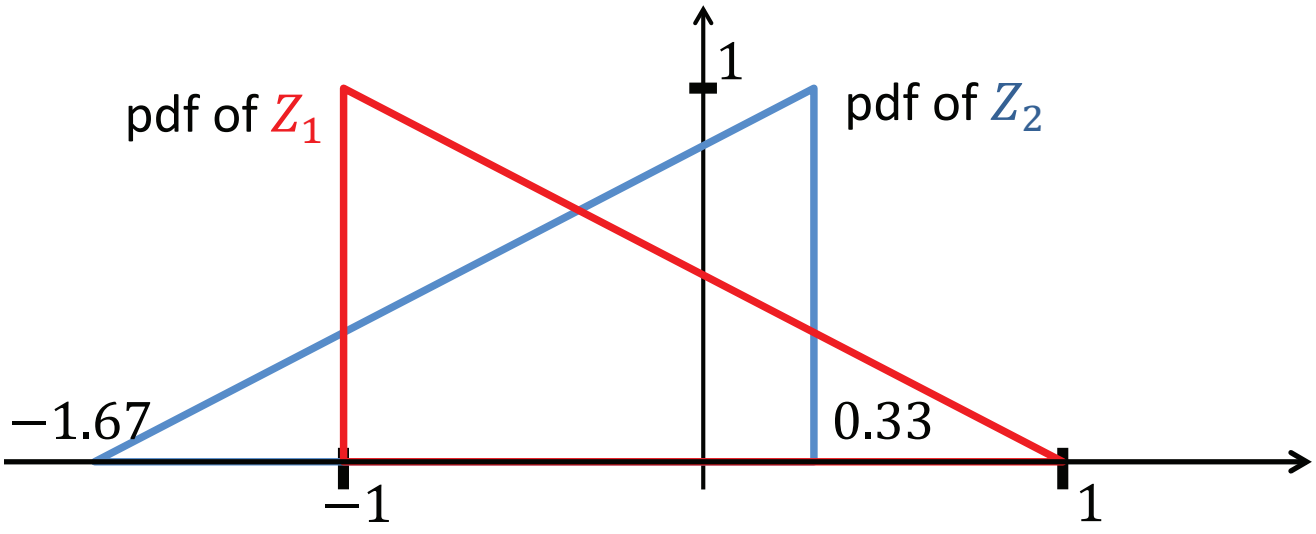
Design 2: uncertain response Z_2



Which design is less uncertain, safer?

Concern about upper tail (displacement, stress, cost)

Details



same mean (-0.33) and std. dev. (0.87)

$\text{Prob}(Z_1 > 0) = 0.25$ (better) and $\text{Prob}(Z_2 > 0) = 0.31$

Failure probability doesn't account for **magnitude** of exceedance

Superquantile risk

Risk assessment in finance: Rockafeller & Uryasev '00, '02 (CVaR); Acerbi & Tasche '02 (exp. shortfall); Föllmer & Schied '04 (AVaR)

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$R_\alpha(Z) =$ average of $(1 - \alpha)100\%$ worst outcomes of Z

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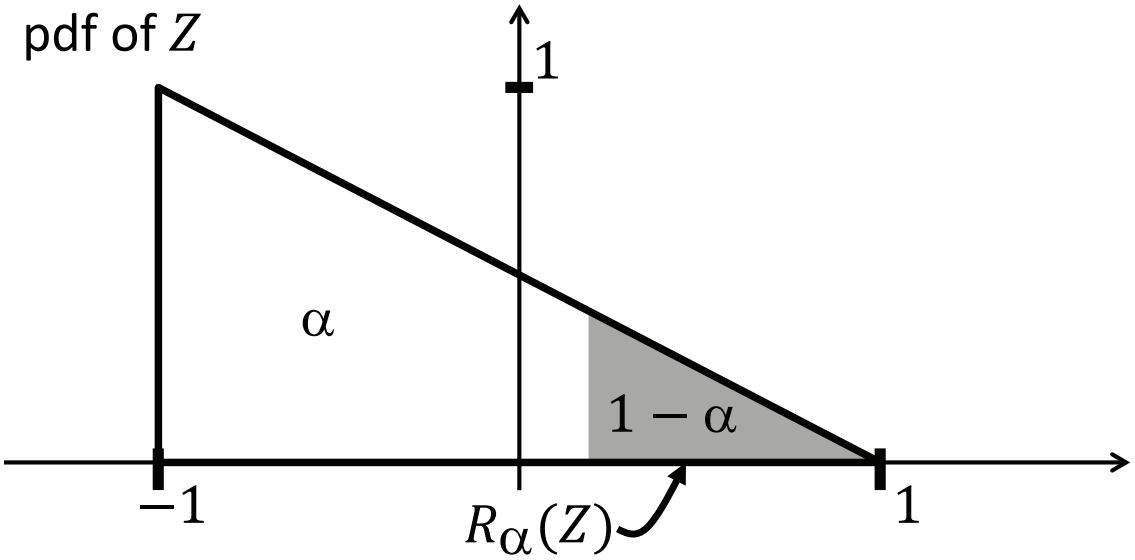

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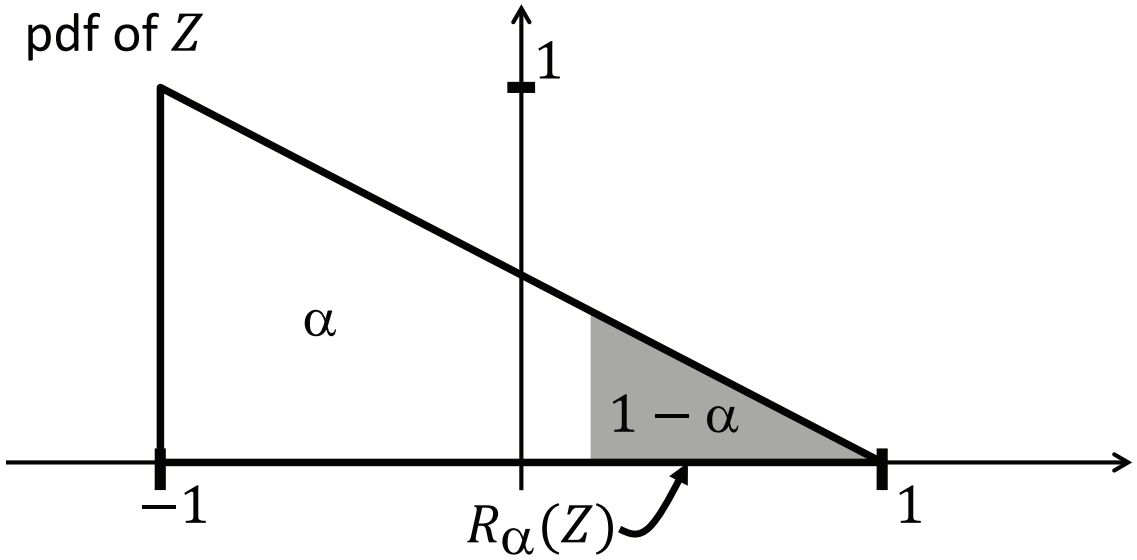


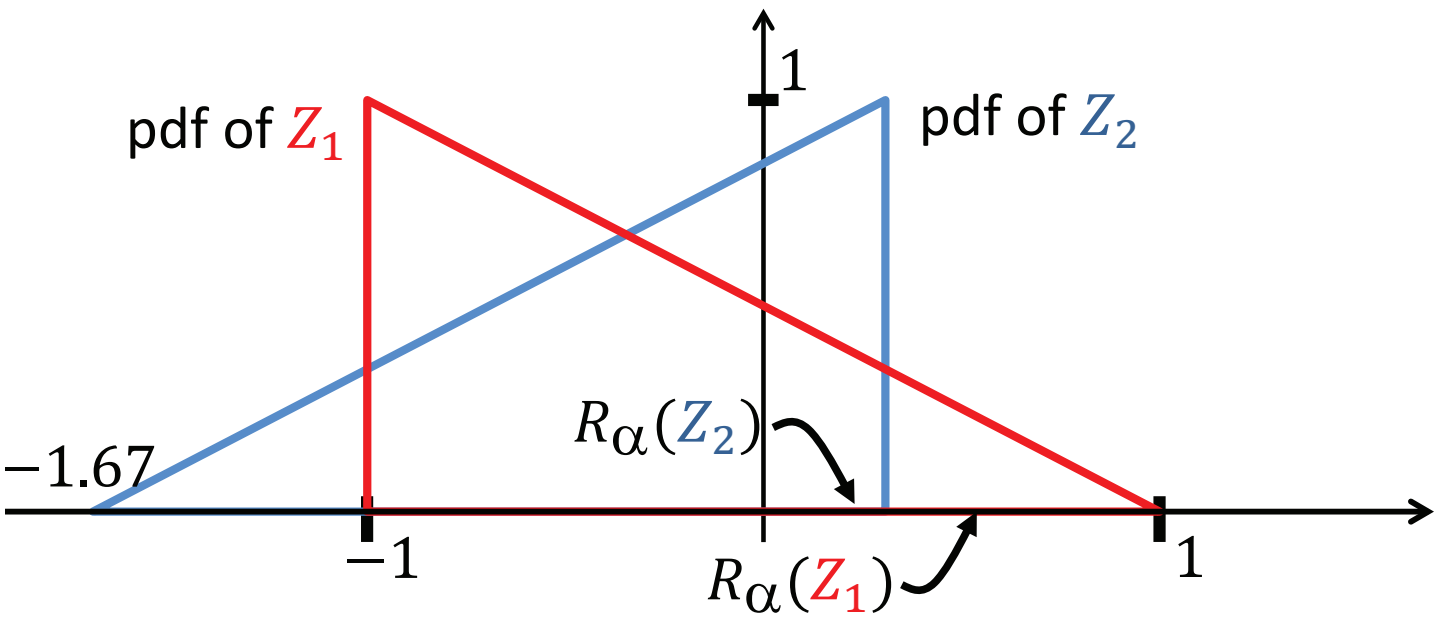
$R_0(Z) = \mathbb{E}[Z]$; $R_1(Z)$ = worst outcome of Z

Z_1 safely below Z_2 when $R_\alpha(Z_1) \leq R_\alpha(Z_2)$

Return to triangular example

Design 1: uncertain response Z_1

Design 2: uncertain response Z_2



Averages of worst 10% outcomes:

$R_{0.9}(Z_1) = 0.58$ and $R_{0.9}(Z_2) = 0.28$ (better)

Response of Design 2 **safely below** that of Design 1

Advantages of superquantile risk (s-risk)

Modeling considerations:

- adapts to any level of "safety" (can vary α)
- focuses on the "bad" tail (promotes resilience)
- promotes diversification
- connects with dual utility theory
- probes deeper than expected utility theory
- relates to risk-neutral decisions under stochastic ambiguity

Computational considerations:

- preserves convexity (continuity)
- easier to find globally optimal designs and decisions
- when using s-risk,
  - optimization under uncertainty "no harder" than deterministic

Connection: failure probability and superquantile risk

Superquantiles lead to a (best) conservative approximation of failure probability through **buffered failure probability** (Rockafellar & Royset '10, Norton et al. '17, Mafusalov et al. '18):

$$
\begin{aligned}
& R_\alpha\big(g(x,V)\big) \le 0 \\
\iff & \text{buffered failure probability of } g(x,V) \le 1-\alpha \\
\implies & \mathrm{Prob}\big(g(x,V) > 0\big) \le 1-\alpha
\end{aligned}
$$

Constraints on s-risk can be reinterpreted in probabilistic terms

Risk-adaptive learning and surrogate building

Response $g(x, V)$ **costly to compute** (high-fidelity simulation)

Leverage approximating responses $h(x, V)$ (low-fidelity simulations)

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Flexibility: $h(x, v)$ vector-valued, possibly $h_j(x, v) = x_j$, etc.

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Example: $\hat{h}(x, v) =$ lower-level surrogate and $f\big(h(x, v)\big) =$

$$a_0 + a^\top x + c^\top v + b_0 \hat{h}(x, v) + \bar{a}^\top x \hat{h}(x, v) + \bar{c}^\top v \hat{h}(x, v) + b[\hat{h}(x, v)]^2$$

Finding f amounts to finding coefficients $a_0, a, \bar{a}, b_0, b, c, \bar{c}$

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Notation: $Y = g(x, V)$, $X = h(x, V)$; view x as "random" over design space (set-based design)

Risk-adaptive learning and surrogates (cont.)

Response quantity: random variable Y (high-fidelity simulation)

Approximations: random vector X (low-fidelity simulations)

Find f such that $R_\alpha(Y) \leq R_\alpha\big(f(X)\big)$

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But using what measure of error? **Least-squares will not do**

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Superquantile regression possible (but not discussed here)
(Rockafellar, Royset, Miranda '14)

Risk-adaptive learning algorithm

For simplicity, $f(X) = c_0 + c^\top X$, with $c \in \mathbb{R}^k$

Two-step algorithm:

1. Solve $\min_{c \in \mathbb{R}^k} \left\{ c^\top \mathbb{E}[X] + R_\alpha(Y - c^\top X) \right\} + \lambda \|c\|_1$
2. Set $c_0 = R_\alpha(Y - c^\top X)$

Step 1 (Residual risk minimization)

convex problem; scalable

problem size is data independent

resembling problem in SVM

Step 2 (s-risk computation)

either 1D convex problem or sorting (quick)

Rockafellar & Royset '15a; Royset, Bonfiglio, Vernengo, Brizzolara '17

Theoretical results

Conservative surrogate on training data:
For $\alpha \in (0,1)$ and (c_0, c) computed by risk-adaptive learning,

$$R_\alpha(\tilde{Y}) \leq R_\alpha(c_0 + c^\top \tilde{X})$$

with $(\tilde{X}, \tilde{Y})$ distributed according to training data

Rockafellar, Uryasev, Zabarankin '08; Rockafellar & Royset '15a

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Consistency:
For $\alpha \in (0,1)$ and (c_0, c) computed by risk-adaptive learning,

$$\mathcal{R}_\alpha(Y) \leq \mathcal{R}_\alpha(c_0 + c^\top X) \text{ in the limit as training size } \to \infty$$

with (X, Y) having the actual (true) distribution

Rockafellar, Uryasev, Zabarankin '08; Rockafellar & Royset '15a

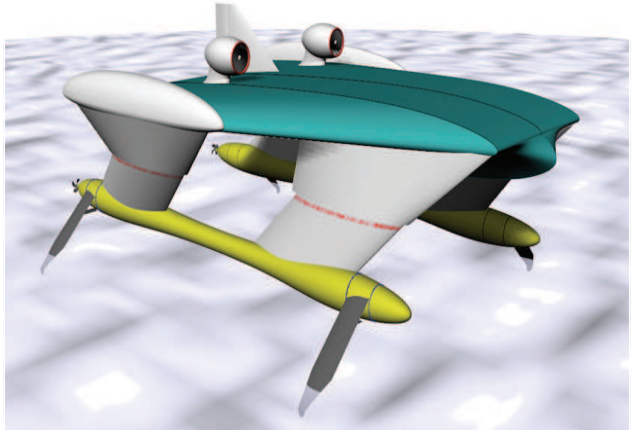
Multi-disciplinary 3D hydrofoil design

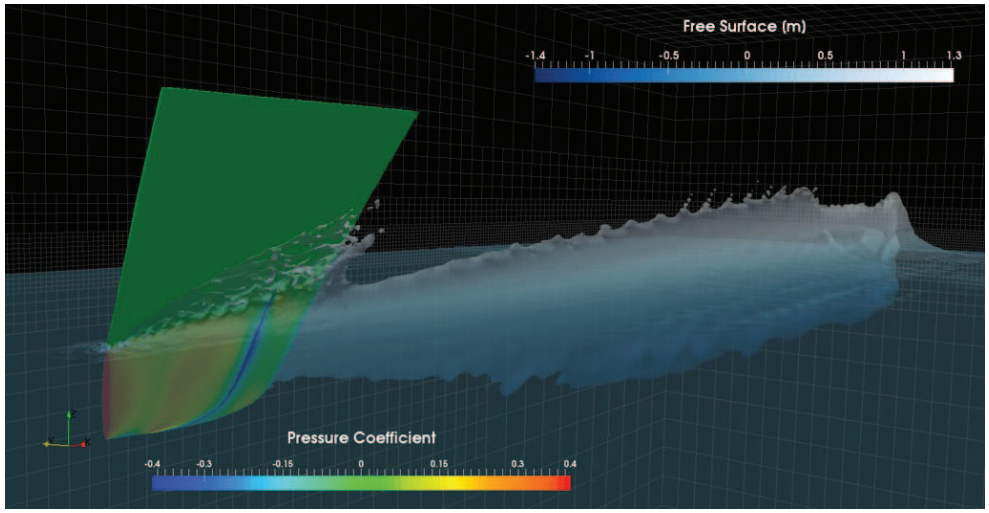
Surface-piercing super-cavitating hydrofoil

17 design variables; 5 uncertain parameters

Quantities of interest: hydrodynamical and structural

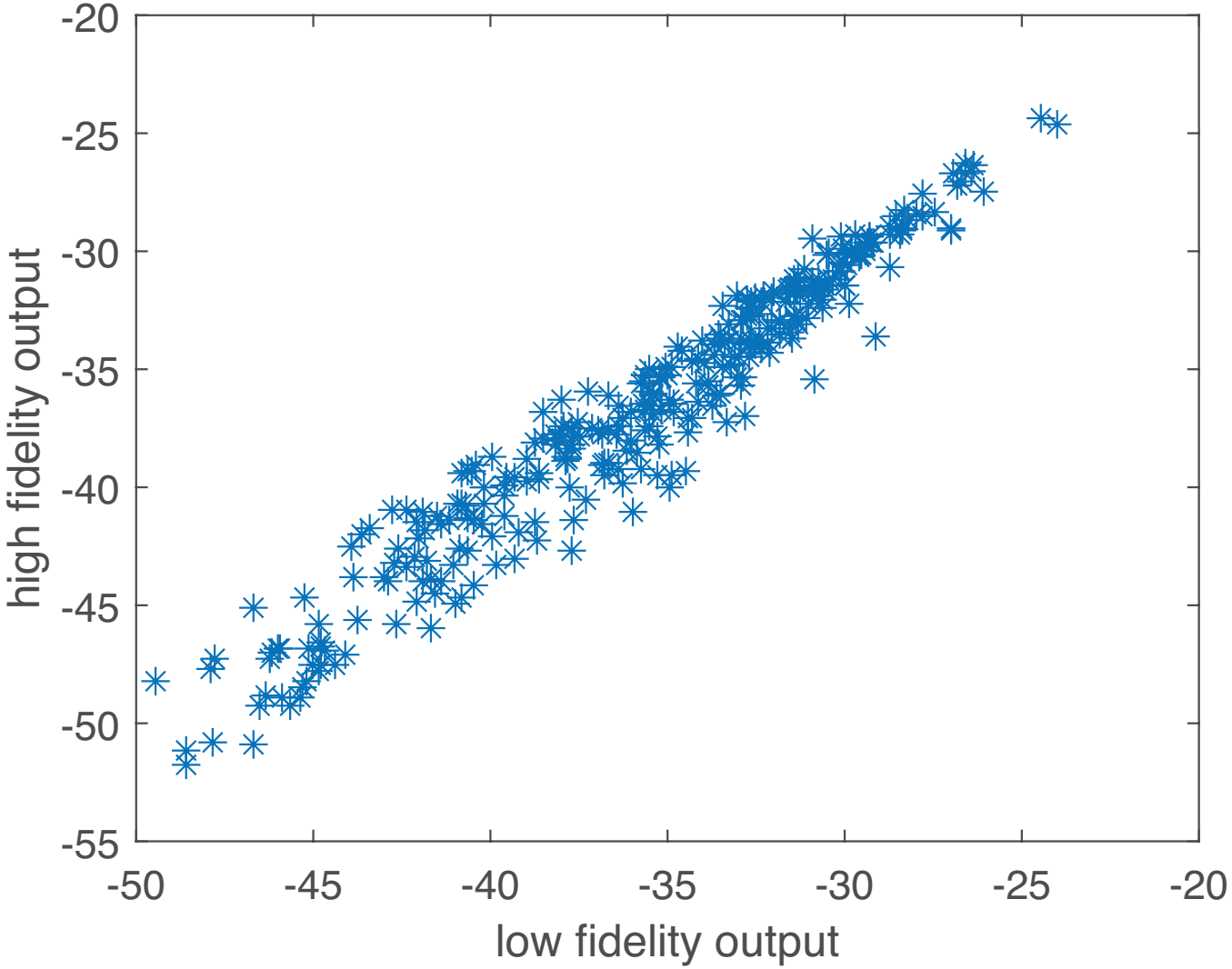
308 high-fidelity 3D URANSE solves
3063 high-fidelity 3D FEM solves
19830 low-fidelity 3D URANSE solves and 3D FEM solves





Bonfiglio & Royset '18

Risk-adaptive learning of lift force

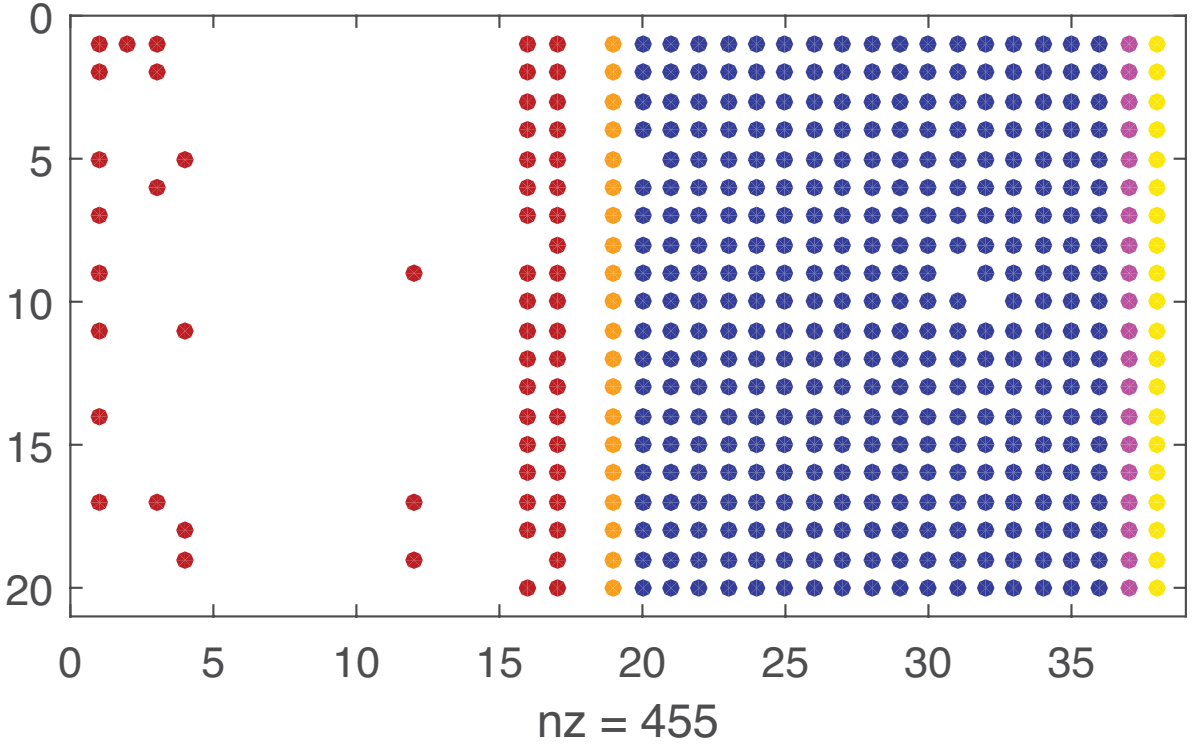


Accurate predictions possible

Risk-adaptive learning of lift force (cont.)

Surrogate has 1+38 coefficients to be learned

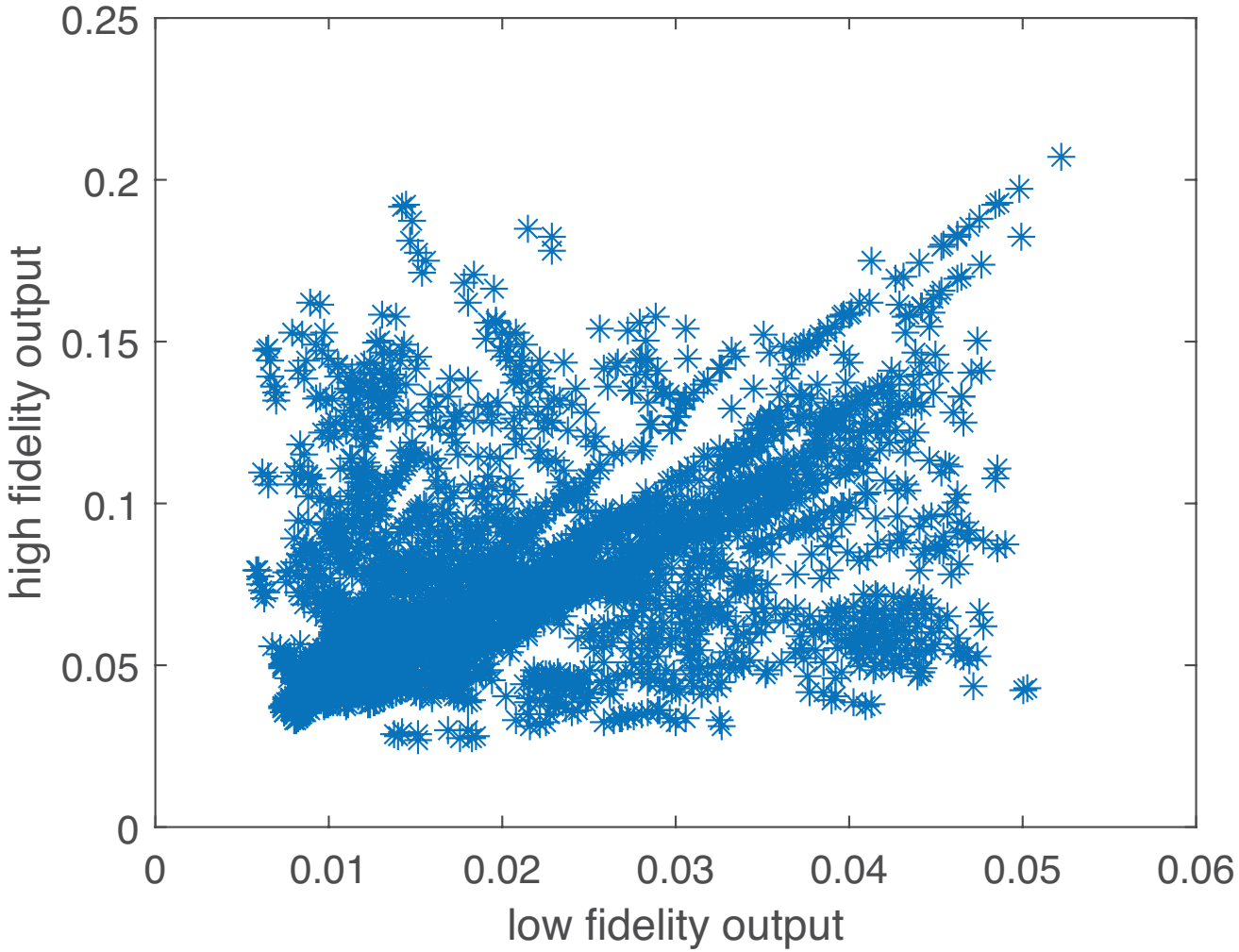
Sparsity (model selection) across 20 surrogates:



Red, gray, orange, blue, pink, and yellow colors correspond to a, c, b_0, $\bar{a}$, $\bar{c}$, and b, respectively

$$a_0 + a^\top x + c^\top v + b_0 \hat{h}(x, v) + \bar{a}^\top x \hat{h}(x, v) + \bar{c}^\top v \hat{h}(x, v) + b[\hat{h}(x, v)]^2$$

Risk-adaptive learning of displacement

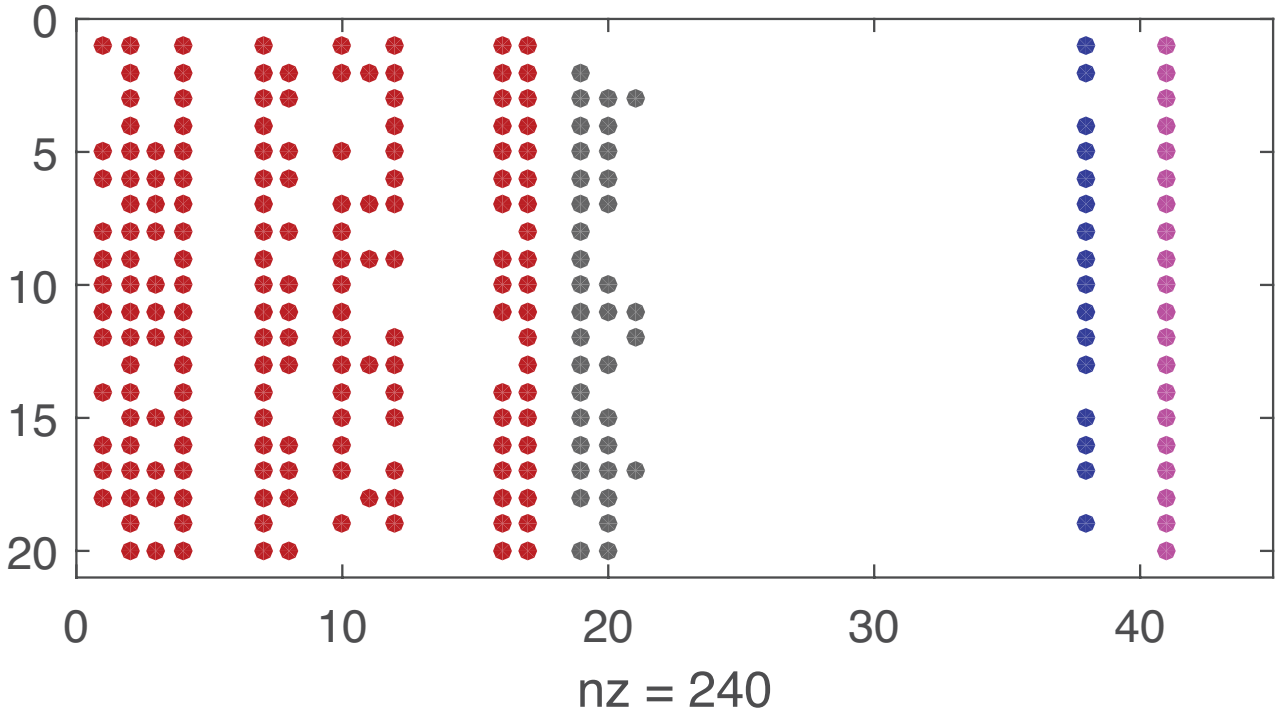


Poor correlation between low- and high-fidelity simulations

Risk-adaptive learning of displacement (cont.)

Surrogate has 1+44 coefficients to be learned

Sparsity (model selection) across 20 surrogates:



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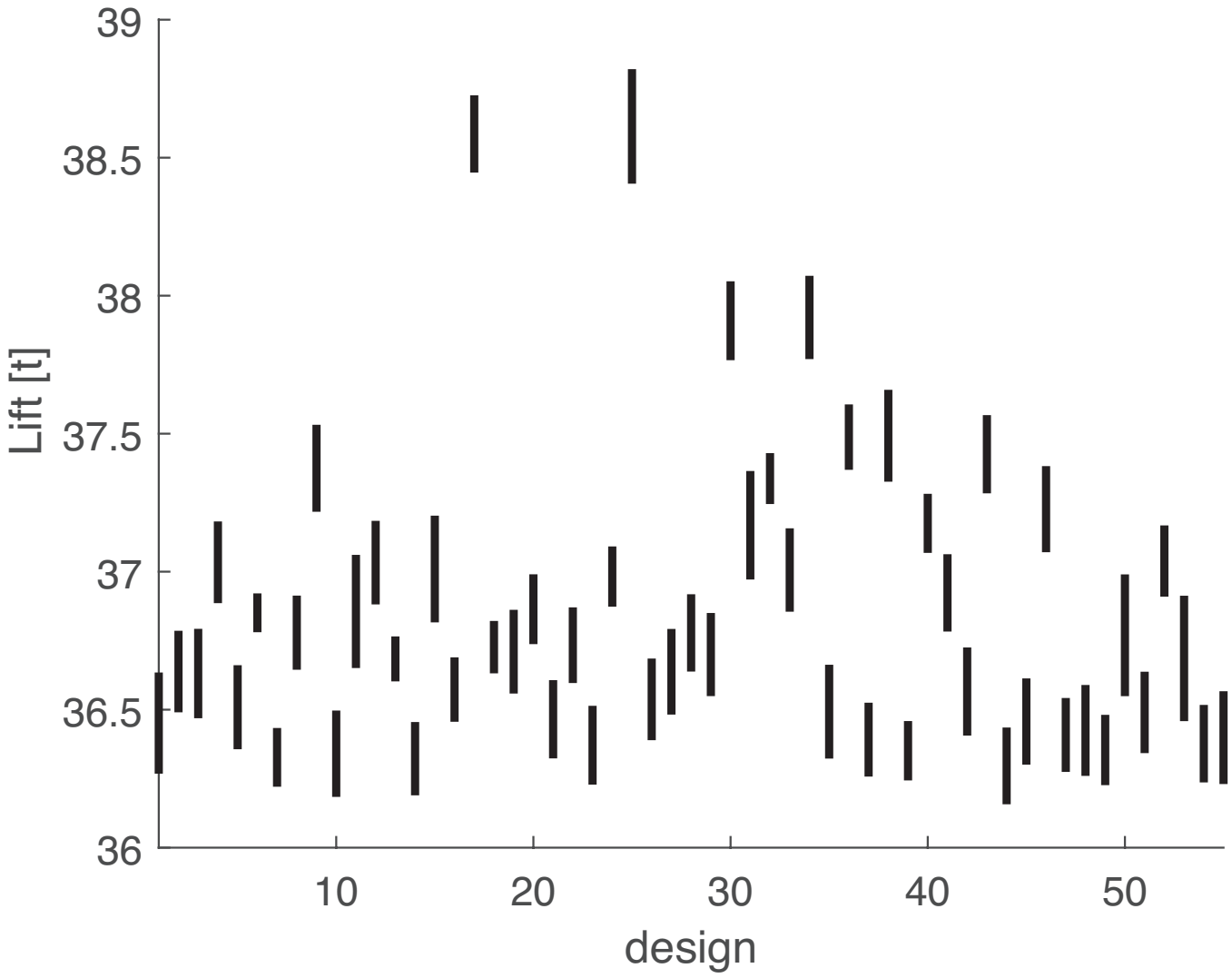
Uncertainty in surrogates: lift

Not standard deviation, but superquantile deviation!



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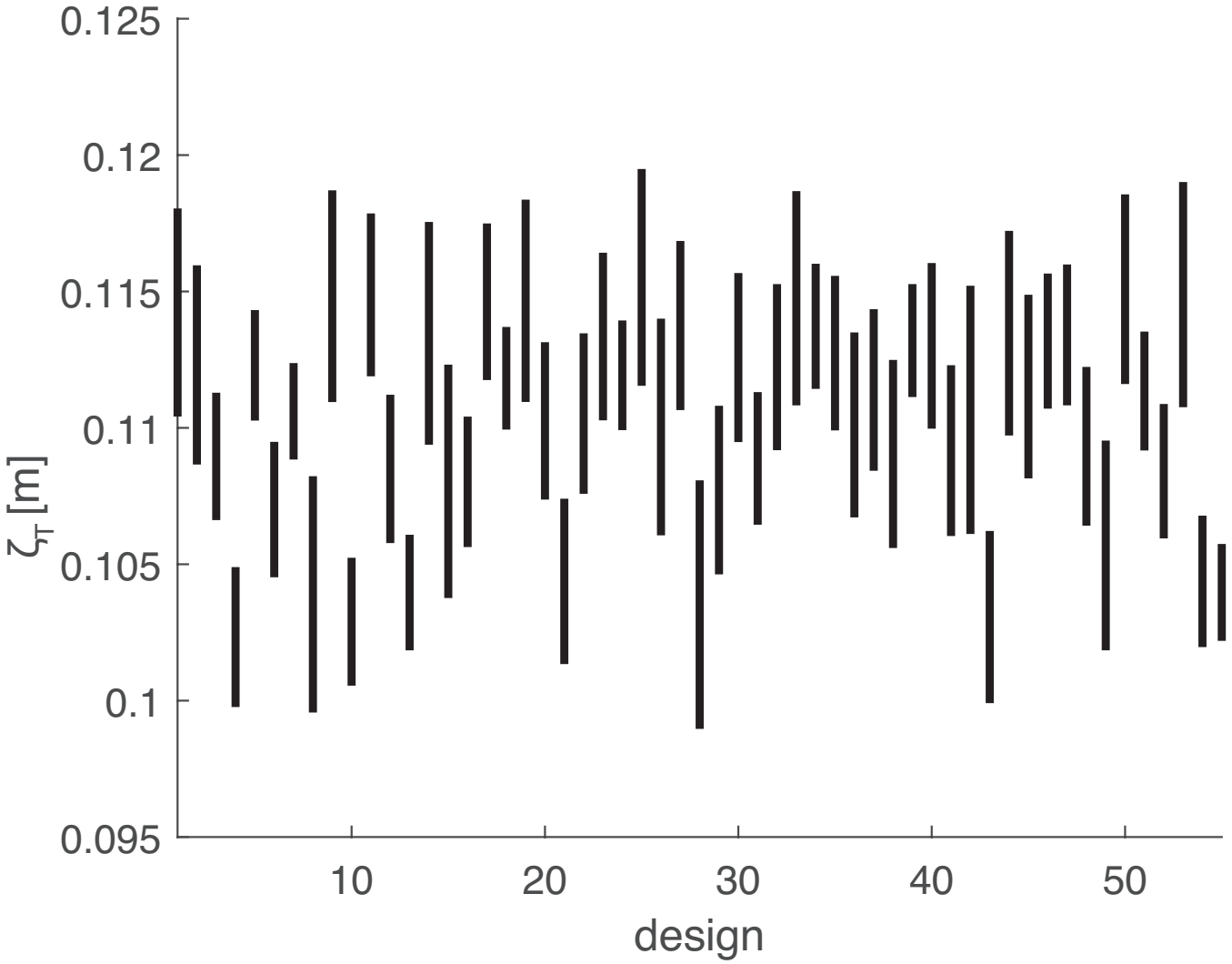
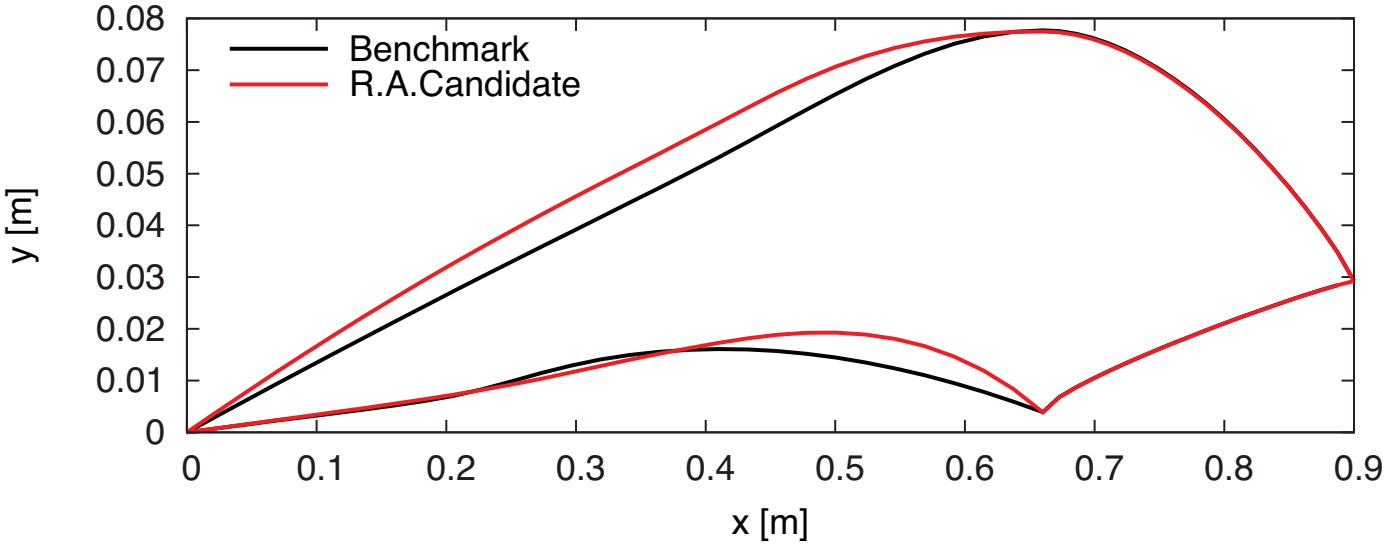


Poor low-fidelity: uncertain surrogates, **but still conservative**

Impact in multi-disciplinary 3D hydrofoil design



	displ. [m]	drag/lift	lift [t]	stress margin [MPa]
Prediction	0.109	0.139	36.8	−142
Actual	0.060	0.130	37.7	−410
Benchmark	0.097	0.132	35.3	−294

References

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R.T. Rockafellar and J.O. Royset, 2018, Superquantile/CVaR Risk Measures: Second-Order Theory, *Annals of Operations Research*, 262(1):3-28.

Bonfiglio & Royset, 2018, Multi-Disciplinary Risk-Adaptive Set-Based Design of Super-Cavitating Hydrofoils, in review.

See http://faculty.nps.edu/joroyset for papers and code