Stochastic Dominance of Lithuanian Pension Funds

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Market structure of lithuanian pension funds

Currently there are ninteen IInd pillar pension funds managed by fife companies operating in Lithuania. By investment strategy, PFs are arranged into four categories depending on the percentage of stocks in the investment:

- Conservative funds (0% stock), CF;
- Small stock funds (less than 30% of stock), SF;
- Medium stock funds (less than 70% of stock), MF;
- Stock funds (up to 100% stock), ST.

Table: Funds market structure (2018 First Quarter)

Fund type	Notation	Number of funds available	Number of participants		Value of managed assets	
			citizens	%	€ millions	%
Conservative	CF	5	104,237	8.52	219.75	10.22
Small stock	SF	4	290,822	23.78	555.76	25.84
Medium stock	MF	6	631,818	51.65	1,115.44	51.86
Stock	ST	4	196,258	16.05	259.78	12.08
Total		19	1,223,135		2,150.73	

From Table 1 we can see that the most popular group is medium stock with 51.65% of participants and the market capitalization of 51.86%.

Where the problem comes from?

We considered a risk-neutral and risk-averse multistage stochastic optimization models, which were used to plan a long-term pension accrual.
The model focuses on three important decision making moments: selection of pension fund at beginning of career of a citizen (first stage decision variable), reselection after one year (second stage decision variable), and then after forty years (third stage decision variable).

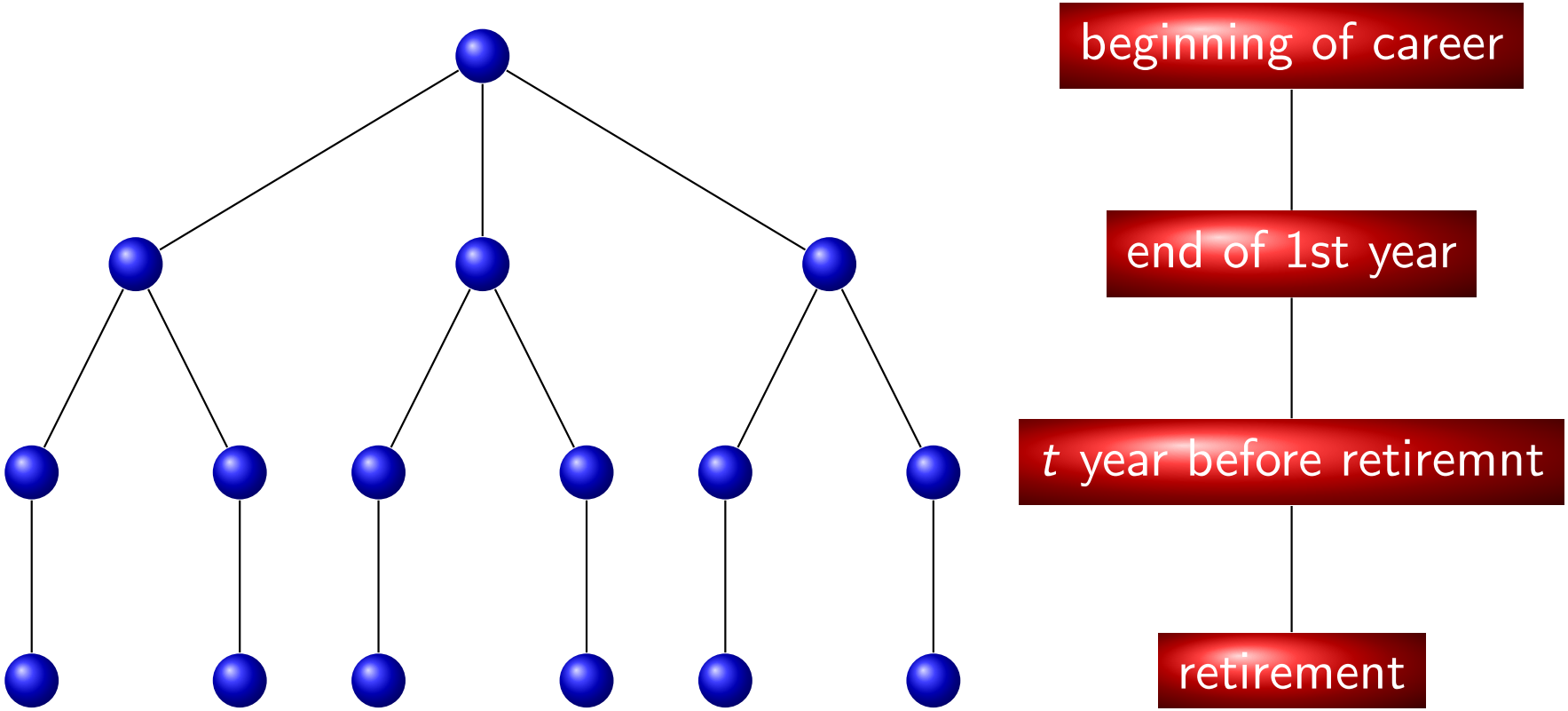

Figure: An example of scenario tree structure adopted to model the uncertainty

The objective of the model is the maximization of the revenues according to the best funds choices in the three stages in life. The information structure can be described in the form of a *scenario tree* $\mathscr{T}$ where at each stage t there is a discrete number of nodes $|n^t|$ where a specific realization of the uncertain parameters takes place. There are H levels (stages) in the tree, that correspond to specific time periods. The final $|n^H|$ nodes are called the leaves.

Parameters:

γ,	fixed parameter dependent on accrual plan;
$f_{n,i}^{(t)}$,	returns of fund i in node n at stage t;
$r_n^{(t)}$,	inflation rate in node n at stage t;
$c_i^{(t)}$,	costs of fund i (transactions, yearly fees) in stage t;
$e_i^{(t)}$,	extra cost paid at stage t if the fund is changed with respect to stage $t-1$;
p_n,	probability of node $n \in \mathscr{N}$;
$\pi_{a(n),n}$,	conditional probability of the random process at node n,given its history up to the ancestor node $a(n)$;
$g_n^{(t)}$,	increment of salary at node n in stage t;
$S_n^{(t)}$,	salary at node n in stage t;
$q^{(t)}$,	number of years in the period t between stage t and $t+1$;
$A_{n,i}^{(t)}$,	accumulated sum during the previous period at stage t of fund i in node n.

Decision variables:

$x_i^{(1)} \in \{0;1\}$, first-stage decision variable: $x_i^{(1)} = 1$ if fund i is selected at the first stage, 0 otherwise;

$x_{n,i}^{(j)} \in \{0;1\}$, decision variable at stage $t = 2,3$: $x_{n,i}^{(t)} = 1$ if fund i is selected at node n at stage t, 0 otherwise;

$y_{n,i}^{(j)} \in \{0;1\}$, decision variable at stage $t = 2,3$: $y_{n,i}^{(t)} = 1$ if fund i selected at node n at stage t is different from the one selected at stage $t-1$, 0 otherwise;

Notice that the accumulated sum $A_{ni}^{(t)}$ for each fund $i = 1, \ldots, n$ during period t is the solution of the following system of difference equations:

$$z(j) = (1 + f_{n,i}^{(t)} - r_n^{(t)}) \cdot z(j-1) + S_n^{(t)}(j) \cdot \gamma \quad (1)$$

$$S_n^{(t)}(j) = (1 + g_n^{(t)}) \cdot S_n^{(t)}(j-1), \quad (2)$$

where z denotes accumulated sum at any moment j of period t and $S^{(t)}(j)$ denotes salary at moment j. $S^{(t)}(0)$ denotes initial salary in that period. Initial accumulate sum $z(0)$ in first period is equal to 0, since no money is transferred to the fund.

The multistage stochastic risk neutral formulation of the problem considered is:

$$\max F = -\sum_{i=1}^{I} c_i^{(1)} x_i^{(1)} + \gamma S^{(1)} + \sum_{t=2}^{3} \sum_{n \in \mathcal{N}^{(t)}} p_n \left[\sum_{i=1}^{I} \left(A_{ni}^{(t)} x_{a(n)i}^{(t-1)} - c_i^{(t)} x_{ni}^{(t)} - e_i^{(t)} y_{ni}^{(t)} \right) \right] + \sum_{n \in \mathcal{N}^{(4)}} p_n \left[\sum_{i=1}^{K} \left(A_{ni}^{(4)} x_{a(n)i}^{(3)} \right) \right] \tag{3}$$

s.t.

$$\sum_{i=1}^{I} x_{ni}^{(t)} = 1, \qquad n \in \mathcal{N}^{(t)},\ t = 1,2,3 \tag{4}$$

$$y_{ni}^{(t)} + x_{a(t)i}^{(t-1)} \leq 1, \qquad i = 1,\ldots,I,\ n \in \mathcal{N}^{(t)}, \tag{5}$$

$$y_{ni}^{(t)} - x_{ni}^{(t)} \leq 0, \qquad i = 1,\ldots,I,\ n \in \mathcal{N}^{(t)}, \tag{6}$$

$$-x_{a(n)i}^{(t-1)} + x_{ni}^{(t)} - y_{ni}^{(t)} \leq 0, \qquad i = 1,\ldots,I,\ n \in \mathcal{N}^{(t)}, \tag{7}$$

$$x_{ni}^{(t)} \in \{0;1\}, \qquad i = 1,\ldots,I,\ n \in \mathcal{N}^{(t)},\ t = 1,2,3, \tag{8}$$

$$y_{ni}^{(t)} \in \{0;1\}, \qquad i = 1,\ldots,I,\ n \in \mathcal{N}^{(t)}.\ t = 2,3, \tag{9}$$

The objective function expresses the maximization of the profits over the three periods given by the difference between the revenues and the transaction costs taking into account of the extra fee paid for changing the fund during stages.

The risk neutral model maximizes the expected total profit along the planning horizon determining the optimal decision variables. However it does not take into account the possibility of a very low profit realization in some scenarios. In order to take into account this, in this research we consider a risk averse strategy which include a suitable risk measure, namely nested multistage *CVaR* which iteratively solves a convex combination of performance and risk in the last stage, using it as a performance measure from the previous stage.

We introduce now:

$d_n^{(t)}$	,	auxiliary variable in node n at stage t;
$\eta_{a(n)}^{(t)}$	,	auxiliary variable at stage t deriving from the ancestor node $a(n)$;
F_n	,	the profit in node n;
$F^{(t)}$	,	the profit at stage t;
$\alpha^{(t)}$	,	the confidence level in the tradeoff between performance and risk at stage t;
$\rho^{(t)}$	,	the weighting factor in the tradeoff between performance and risk at stage t.

At stage four we define:

$$d_m^{(4)} \geq 0,\ d_m^{(4)} \geq \eta_n^{(4)} - F_m,\ n \in \mathscr{N}^{(3)},\ m \in \mathscr{B}(n) \tag{10}$$

then for each node of stage three we define:

$$d_m^{(3)} \geq 0,\ d_m^{(3)} \geq \eta_n^{(3)} - K_m,\ n \in \mathscr{N}^{(2)},\ m \in \mathscr{B}(n) \tag{11}$$

where

$$K_n = F_n + (1-\rho^{(4)})\Big(\sum_{m\in\mathscr{B}(n)} \pi_{n,m} F_m\Big) + \rho^{(4)}\left(\eta_n^{(4)} - \frac{1}{\alpha^{(4)}} \sum_{m\in\mathscr{B}(n)} \pi_{n,m} d_m^{(4)}\right),\ n \in \mathscr{N}^{(3)}\ .$$

The term $(1-\rho^{(4)})(\sum_{m\in\mathscr{B}(n)}\pi_{n,m}F_m)+\rho^{(4)}\left(\eta_n^{(4)}-\frac{1}{\alpha^{(4)}}\sum_{m\in\mathscr{B}(n)}\pi_{n,m}d_m^{(4)}\right)$ represents the objective function at stage four conditioning to reaching node $n\in\mathscr{N}^{(3)}$ at stage three.

Then for each node of stage two we define

$$d_n^{(2)} \geq 0,\ d_n^{(2)} \geq \eta_{a(n)}^{(2)} - L_n,\ n \in \mathscr{N}^{(2)}, \tag{12}$$

where

$$L_n = F_n + (1-\rho^{(3)})(\sum_{m \in \mathscr{B}(n)} \pi_{n,m} K_m) + \rho^{(3)} \left(\eta_n^{(3)} - \frac{1}{\alpha^{(3)}} \sum_{m \in \mathscr{B}(n)} \pi_{n,m} d_m^{(3)} \right),\ n \in \mathscr{N}^{(2)}.$$

The term $(1-\rho^{(3)})(\sum_{m \in \mathscr{B}(n)} \pi_{n,m} K_m) + \rho^{(3)} \left(\eta_n^{(3)} - \frac{1}{\alpha^{(3)}} \sum_{m \in \mathscr{B}(n)} \pi_{n,m} d_m^{(3)} \right)$, defines the objective function at stage three conditioned to reaching node $n \in \mathscr{N}^{(2)}$ in stage two.

Finally the objective function on the first stage is given by

$$F^{(1)} + (1-\rho^{(2)})\Big(\sum_{n\in\mathcal{N}^{(2)}} \pi_{1,n} L_n\Big) + \rho^{(2)}\Big(\eta^{(2)} - \frac{1}{\alpha^{(2)}} \sum_{n\in\mathcal{N}^{(2)}} \pi_{1,n} d_n^{(2)}\Big) \ . \qquad (13)$$

α-Stable Distribution

α-stable distribution belongs to the models for heavy tailed data.

It is characterized by four parameters: α – index of stability, σ – scale parameter, β – skewness parameter, μ – location parameter.

Shortly, the notation $S_\alpha(\sigma,\beta,\mu)$ is used to denote the class of stable laws. Generally, the characteristic function $\phi_X(t)$ of a random variable X, which is distributed by α-stable law, is

$$\phi_X(t) = \begin{cases} \exp\left\{-\sigma^\alpha |t|^\alpha \left(1 - i\beta \left(\tan \frac{\pi\alpha}{2}\right) (\text{sign } t)\right) + i\mu t\right\}, \alpha \neq 1; \\ \exp\left\{-\sigma^\alpha |t|^\alpha \left(1 - i\beta \frac{\pi}{2} (\text{sign } \ln |t|)\right) + i\mu t\right\}, \quad \alpha = 1. \end{cases} \tag{14}$$

The index of stability α determines the rate at which the tails decay. If $\alpha = 2$ the characteristic function in given equation reduces to the characteristic function of the normal distribution. If $\alpha \leq 1$, then the expectation of random variable cannot be defined.

Mixed α-Stable Distribution

Mixed α-stable distribution was applied for modeling the financial data. The additional parameter $p \in [0,1]$ is included to model zero daily returns with a certain probability, i.e.

$$X_{mix} = \begin{cases} 0, & p < u\,; \\ S_\alpha(\sigma,\beta,\mu), & p \geqslant u\,; \end{cases} \tag{15}$$

where u is uniform random variable $u \sim U(0,1)$.

The probability density function of a mixed α-stable distribution is given as

$$f_{mix}(x) = p \cdot \delta(x) + (1-p) \cdot f_\alpha(x) \tag{16}$$

where $f_\alpha(x) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \phi_X(t) \cdot e^{-ixt} \mathrm{d}t$ is the probability density function of an α-stable distribution expressed through its characteristic function and $\delta(x)$ is the Dirac delta function.

In this case cumulative distribution function or CDF is

$$F_{mix}(x) = (1-p) \cdot F_{\alpha}(x, \alpha_0, \beta_0, \mu_0, \sigma_0) + p \cdot \varepsilon(x), \qquad (17)$$

where $F_{\alpha}(x, \alpha_0, \beta_0, \mu_0, \sigma_0)$ is CDF of α-stable distribution, where $\varepsilon(x) = \begin{cases} 0, & x \leqslant 0 \\ 1, & x > 0 \end{cases}$, is the CDF of the degenerate distribution, the vector pf parameters $\alpha_0, \beta_0, \mu_0, \sigma_0$ is estimated with nonzero data.

In our research we use daily fund returns to estimate the parameters of α-stable distribution, however, in the model proposed we use yearly values of fund return.
To solve this problem we suggest if $X_1, X_2, \ldots, X_n$ are i.i.d. $S_\alpha(\sigma, \beta, \mu)$, then

$$\sum_{i=1}^{n} X_i \stackrel{d}{=} \begin{cases} n^{1/\alpha} X_1 + (\mu - \lambda)(n - n^{1/\alpha}), & \alpha \neq 1, \\ nX_1 + \dfrac{2}{\pi}\sigma\beta n \ln n, & \alpha = 1. \end{cases} \tag{18}$$

It is assumed to set up $n = 252$ equal to number of working days in year, $\lambda = \beta\sigma \tan(\pi\alpha/2)$ is a correction constant of Nolan parametrization.

PF	alpha	beta	mu	sigma
AVIVA EURO	1.343	-0.083	0.00037	0.00096
AVIVA EUROextra	1.645	-0.539	0.00209	0.00481
AVIVA EUROplius	1.672	-0.476	0.00118	0.00275
INVL_EXTREMO	1.409	-0.118	0.00699	0.02483
INVL_MEDIO	1.516	-0.177	0.00637	0.01941
INVL_MEZZO	1.405	-0.008	0.00222	0.00712
INVL_STABILO	1.223	-0.190	0.00223	0.00240
LUMINOR1	1.455	0.018	0.00170	0.00261
LUMINOR2	1.411	-0.270	0.00372	0.00749
LUMINOR3	1.450	-0.225	0.00417	0.01069
SEB1	1.261	0.142	0.00106	0.00424
SEB2	1.494	-0.225	0.00484	0.01445
SEB3	1.503	-0.152	0.00698	0.02548
SWED1	1.384	0.247	0.00047	0.00088
SWED2	1.749	-0.778	0.00157	0.00289
SWED3	1.723	-0.760	0.00229	0.00491
SWED4	1.737	-0.749	0.00285	0.00718
SWED5	1.752	-0.633	0.00351	0.01057

Table: Results of estimation of alpha-stable parameters

Inflation dynamics model

In our research, the discrete-time Markov-Switching AutoRegressive (MS-AR) model is used to create scenario for inflation dynamics.

For our particular application, MS-AR model has two components: $\{r_t\} \in \Re$ denotes inflation dynamics in time, $\{R_t\} \in \{1, \ldots, R\}$ represents the regimes corresponding to different states of money market.

Salary increment

The estimated characteristics of historical salary increment are used to generate the possible values of salary scenarios.

Table: Empirical statistics of salary increment in Lithuania

mean	0.00507143
stdev	0.06424648
skewness	-1.78825
kurtosis	3.849164

Numerical Results of Multi stage stochastic programing

	α \ ρ	0^*	0.05	0.1	0.25	0.5	0.75	1
$t^{(3)} = 0$	0.01	1	1	1	1	1	1	1
	0.05	1	1	1	1	1	1	1
	0.1	1	1	1	1	1	1	1
$t^{(3)} = 1$	0.01	1	1	1	1	1	1	4
	0.05	1	1	1	1	1	1	4
	0.1	1	1	1	1	1	1	4
$t^{(3)} = 3$	0.01	1	1	1	1	1	1	4
	0.05	1	1	1	1	1	1	4
	0.1	1	1	1	1	1	1	4
$t^{(3)} = 5$	0.01	1	1	1	1	1	4	4
	0.05	1	1	1	1	1	1	4
	0.1	1	1	1	1	1	1	4
$t^{(3)} = 7$	0.01	1	1	1	1	1	4	4
	0.05	1	1	1	1	1	4	4
	0.1	1	1	1	1	1	1	4

First stage solution for investor with low weighting factor ρ (0 or 0.5, investor threats variability of fund value much less important than profit) suggests to choose fund number $i = 1$ (INVL1, conservative fund). If the person is risk avoiding ($\rho = 1$ or 0.75) then fund number $i = 4$ (DANSKE1, conservative fund) should be selected according to the results of the experiment. It must be noticed that in practice $\alpha \leq 0.1$ is usually used, because in such case the upper bound of 10% worst cases over all history is treated as risky.

It is clear from Table 29 that CVaR level α has very weak influence to the solution (with exception of cases $\rho = 0.75$ and $t^{(3)} = 5,7,10,15$). This may happen if selected fund has very strong stochastic tail dominance over all other funds compared.

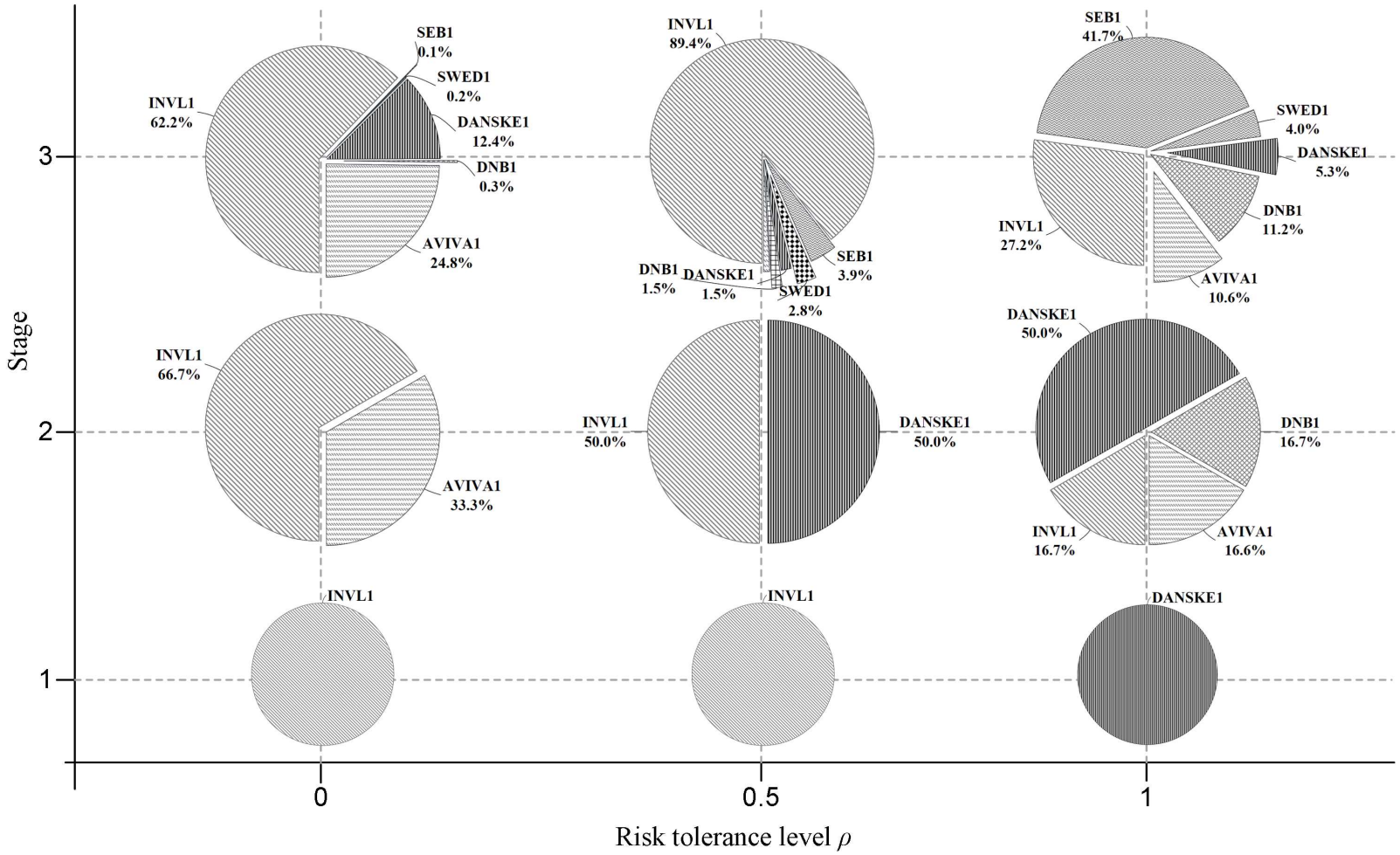

Figure: Selection of fund in first, second and third stages

Risk measures and comparison of funds

	Min	Q1	Median	Q3	Max	Mean	IQR	Stdev	Skewnes	Kurtosis
SWED1	-0.014	-0.0002	0.0005	0.002	0.008	0.0005	0.002	0.002	-1.88	12.1
SEB1	-0.076	-0.0027	0.0013	0.006	0.080	0.0016	0.008	0.012	0.00	12.6
LUMINOR1	-0.079	-0.0010	0.0017	0.004	0.032	0.0015	0.005	0.007	-3.70	41.9
INVL.STABILO	-0.099	0.0000	0.0021	0.004	0.040	0.0014	0.004	0.010	-3.98	39.4
AVIVA.EURO	-0.014	-0.0006	0.0004	0.001	0.017	0.0004	0.002	0.003	1.05	11.2
SWED2	-0.019	-0.0018	0.0013	0.004	0.017	0.0009	0.006	0.005	-0.52	1.3
LUMINOR2	-0.089	-0.0043	0.0031	0.010	0.081	0.0019	0.014	0.017	-0.85	6.2
INVL.MEZZO	-0.092	-0.0044	0.0022	0.009	0.077	0.0016	0.014	0.017	-1.19	8.5
AVIVA.EUROplius	-0.020	-0.0018	0.0009	0.003	0.030	0.0006	0.005	0.005	-0.04	4.3
SWED4	-0.044	-0.0057	0.0022	0.009	0.046	0.0011	0.014	0.012	-0.44	1.4
SWED3	-0.024	-0.0037	0.0018	0.006	0.028	0.0010	0.010	0.008	-0.43	0.9
SEB2	-0.188	-0.0112	0.0039	0.018	0.220	0.0027	0.029	0.033	-0.09	10.7
LUMINOR3	-0.098	-0.0070	0.0038	0.013	0.113	0.0022	0.020	0.022	-0.40	4.4
INVL.MEDIO	-0.293	-0.0139	0.0059	0.023	0.196	0.0037	0.037	0.041	-1.19	10.2
AVIVA.EUROextra	-0.032	-0.0031	0.0016	0.006	0.042	0.0009	0.009	0.008	-0.39	2.4
SWED5	-0.082	-0.0086	0.0030	0.012	0.066	0.0014	0.020	0.017	-0.57	2.2
SEB3	-0.278	-0.0179	0.0061	0.029	0.316	0.0044	0.047	0.054	-0.15	8.0
INVL.EXTREMO	-0.334	-0.0186	0.0068	0.028	0.319	0.0047	0.047	0.057	-0.70	9.2

Figure: Empirical characteristics

Fund	Historical VaR (99%)	Historical CVaR (99%)	Modified VaR (99%)	Modified CVaR (99%)	Downside Deviation (MAR = 0%)	Sortino Ratio (MAR = 0%)	Sharpe (Rf=0%)	Worst Drawdown	Rachev (Rf=0%, p=0.99)
SWED1	-0.008	-0.010	-0.010	-0.010	0.001	0.358	0.251	0.043	1.626
SEB1	-0.032	-0.049	-0.059	-0.059	0.007	0.216	0.139	0.176	0.980
LUMINOR1	-0.017	-0.039	-0.070	-0.070	0.005	0.280	0.209	0.129	1.423
INVL.STABILO	-0.035	-0.063	-0.081	-0.081	0.008	0.180	0.144	0.173	1.762
AVIVA.EURO	-0.005	-0.008	-0.009	-0.009	0.001	0.281	0.161	0.041	0.608
SWED2	-0.013	-0.015	-0.013	-0.016	0.003	0.274	0.183	0.062	1.091
LUMINOR2	-0.047	-0.073	-0.066	-0.066	0.012	0.161	0.115	0.215	1.335
INVL.MEZZO	-0.068	-0.084	-0.077	-0.077	0.012	0.128	0.094	0.242	1.463
AVIVA.EUROplius	-0.013	-0.015	-0.015	-0.015	0.003	0.200	0.133	0.049	0.966
SWED4	-0.034	-0.039	-0.033	-0.040	0.008	0.137	0.096	0.144	1.151
SWED3	-0.022	-0.024	-0.021	-0.026	0.005	0.187	0.128	0.108	1.051
SEB2	-0.089	-0.146	-0.158	-0.158	0.023	0.118	0.081	0.399	0.999
LUMINOR3	-0.070	-0.086	-0.078	-0.078	0.016	0.143	0.100	0.268	1.081
INVL.MEDIO	-0.129	-0.192	-0.204	-0.204	0.030	0.123	0.089	0.505	1.403
AVIVA.EUROextra	-0.025	-0.028	-0.025	-0.027	0.006	0.151	0.106	0.101	1.126
SWED5	-0.051	-0.064	-0.053	-0.058	0.012	0.111	0.079	0.209	1.316
SEB3	-0.166	-0.228	-0.229	-0.229	0.037	0.118	0.081	0.584	0.982
INVL.EXTREMO	-0.189	-0.271	-0.270	-0.270	0.040	0.116	0.082	0.629	1.318

Figure: Risk measures

X \ Y	SWED4	SWED3	SWED2	SWED1	SEB1	SEB2	SEB3	LUMINOR 1	LUMINOR 2	LUMINOR 3	INVL. STABILO	INVL. MEZZO	INVL. MEDIO	INVL. EXTREMO	AVIVA. EURO	AVIVA. EUROplius	AVIVA. EUROextra
SWED5	206	206	208	151	200	208	150	144	153	218	31	143	122	207	214	149	340
SWED4		207	208	202	206	208	151	147	151	220	158	145	142	212	214	150	199
SWED3			134	168	136	209	203	215	194	226	148	91	133	139	214	229	144
SWED2				87	138	213	67	215	116	224	148	148	148	144	145	232	147
SWED1					210	215	138	217	142	170	237	148	144	212	150	250	217
SEB1						193	185	214	163	153	290	148	147	212	127	215	218
SEB2							109	215	153	151	167	330	212	216	160	228	223
SEB3								172	145	18	160	158	211	212	150	154	149
LUMINOR1									148	141	153	156	230	212	150	151	217
LUMINOR2										145	151	147	151	209	267	220	219
LUMINOR3											104	151	210	212	354	223	221
INVL.STABILO												157	208	214	194	223	217
INVL.MEZZO													192	186	148	182	217
INVL.MEDIO														172	204	152	218
INVL.EXTREMO															248	150	150
AVIVA.EURO																280	153
AVIVA.EUROplius																	156

Figure: First order stochastic dominance (empirical)

X \ Y	SWED4	SWED3	SWED2	SWED1	SEB1	SEB2	SEB3	LUMINOR 1	LUMINOR 2	LUMINOR 3	INVL. STABILO	INVL. MEZZO	INVL. MEDIO	INVL. EXTREMO	AVIVA. EURO	AVIVA. EUROplius	AVIVA. EUROextra
SWED5	7	6	9	355	3	10	358	352	358	26	341	350	351	5	11	354	104
SWED4		6	0	0	28	8	355	355	356	24	357	352	355	6	10	357	334
SWED3			358	24	351	4	4	16	14	365	353	340	20	339	10	19	352
SWED2				340	355	6	339	12	350	26	356	354	365	354	359	17	355
SWED1					12	10	350	11	353	353	51	353	354	10	359	20	14
SEB1						15	14	8	40	358	365	355	356	8	365	10	12
SEB2							353	15	357	356	364	338	22	8	364	17	11
SEB3								349	353	113	350	360	20	7	356	353	359
LUMINOR1									356	352	357	355	30	7	357	356	14
LUMINOR2										354	355	360	353	8	74	15	13
LUMINOR3											316	357	4	8	261	14	13
INVL.STABILO												3	3	9	355	14	11
INVL.MEZZO													3	4	359	361	10
INVL.MEDIO														354	12	354	10
INVL.EXTREMO															208	356	337
AVIVA.EURO																43	346
AVIVA.EUROplius																	352

Figure: Second order stochastic dominance (empirical)

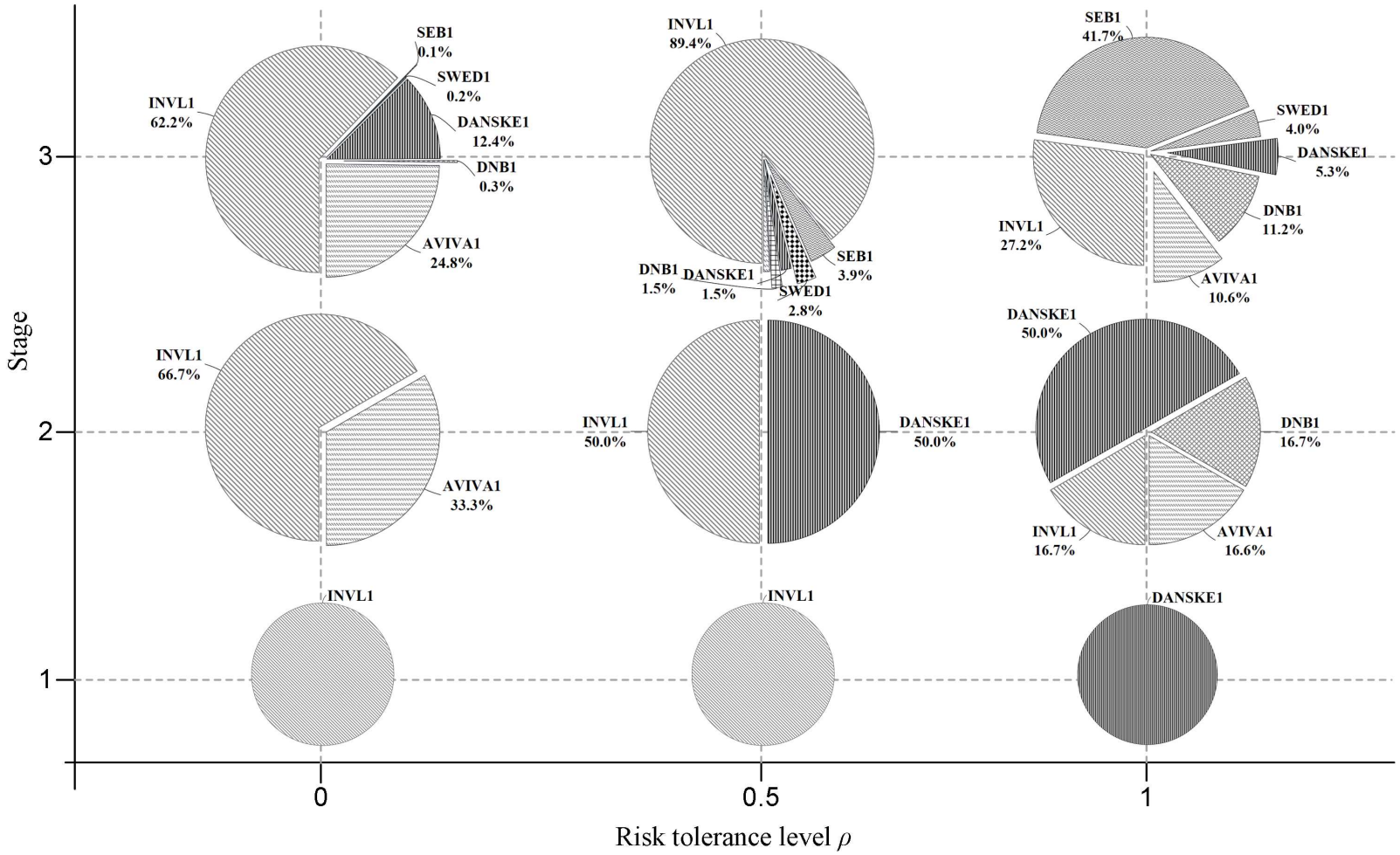

Figure: Selection of fund in first, second and third stages

Conclusions

In this paper we have investigated the possible improvements to be implemented in the IInd pillar of Lithuanian pension system. This topic is highly relevant in Lithuania since those changes are under debate by supervisor of financial markets and the government of Lithuania. The pension scheme is modelled using a risk averse and risk neutral multistage stochastic programming methodology. The study covers the minimum and maximum accumulation plans (respectively, Case A and Case B) for the different sizes of initial salary and the different duration time until retirement that influences the target date.

The numerical results show that in the long term accumulation

The accumulation in Case B allows to achieve approximately double savings in a long period comparing to Case A. Moreover, in minimal accumulation scheme the participant may not exceed necessary fund to cover living expenses over lifetime in retirement. This conclusion may be used to make final political and economical decision at the parliament and government of Lithuania.

Given the model assumptions for a participant of 18 years old in both Case A and Case B considered the switching to conservative accumulation must be done for shorter time as possible. In risk averse case, such compulsory switching, as an improvement of pension system to be concerned, implies approximately 190% loss if compared when such legal regulation is not applied. However, in risk neutral case the

First and second order stochastic dominance confirms (however, not strongly) previous assumptions about dominance of conservative pension funds in respect to others. We are still working on third order SD and parametric SD approach. Analysis of performance measures related to alpha-stable parameters assumes that conservative funds are more or less dominating.