

Workshop on Risk Management Approaches in  
Engineering Applications  
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Organizers: Stan Uryasev, Matthew Norton, Aleksandr Mafusalov,

**Solving multistage stochastic optimization problems with risk measures by  
the progressive hedging algorithm**

Tyrell Rockafellar, University of Washington

The progressive hedging algorithm (Rockafellar/Wets 1991) can solve multistage stochastic programming problems by iteratively decomposing them into multistage deterministic problems, one for each scenario. Price vectors capturing the value of information are generated in this process. So far, however, only objectives of expectation form have been covered. This talk will explain how the method can be extended to minimizing a measure of risk of the random cost instead of just an expectation.

# **Optimal Approximation of Spectral Risk Measures with Application to PDE-Constrained Optimization**

Drew Kouri, Sandia National Laboratory

Many science and engineering applications require the control or design of a physical system governed by partial differential equations (PDEs). More often than not, PDE inputs such as coefficients, boundary conditions, or initial conditions are unknown and estimated from experimental data. Such problems can be formulated as risk-averse optimization problems in Banach space. In this talk, I concentrate on spectral risk measures for PDE-constrained optimization. In particular, I develop an optimal quadrature approximation of spectral risk measures. This approximation is provably convergent and results in a consistent approximation of the original optimization problem. In addition, I show that a large class of spectral risk measures can be generated through the risk quadrangle. I conclude with numerical examples demonstrating these results.

# Optimal Bidding Strategy for Electricity Market Participants Considering Renewable Generation and Price Uncertainties

Kai Pan, University of Florida

Yongpei Guan, University of Florida

As renewable energy has been increasingly penetrating into the power grid system, due to its intermittent nature, both electricity price and renewable generation output are volatile. This brings challenges for independent power producers (IPPs) to manage their generation assets and submit bidding offers with the objective of maximizing their total profits. In this paper, an optimal bidding strategy is derived for IPPs owning coal-fired generators by participating in both day-ahead and real-time markets as a price taker. In the proposed approach, an IPP submits an offer in terms of generation amounts for each operating hour to the day-ahead market, for which a multistage adaptive stochastic optimization setting is explored for submitting real-time market offers for each particular hour as a recourse by taking advantages of the more accurate forecasting of renewable energy output and real-time price as the forecast range shrinks. This proposed strategy is theoretically justified of its significant advantages over existing alternative ones. To improve the computational efficiency, polyhedral structures have been explored to strengthen the corresponding deterministic equivalent formulation. In particular, several families of strong valid inequalities are derived, which are strong enough to provide the convex hull descriptions for the two-period and certain three-period cases. In addition, polynomial time separation algorithms have been derived for the multi-period exponential-sized inequalities so as to speed up the branch-and-cut process. Finally, the numerical studies show the promising future of adapting the proposed strategy and verify the effectiveness of the proposed strong valid inequalities.

## Accelerated gradient sliding for structured convex optimization

Guanghai (George) Lan, Georgia Tech

Yuyuan Ouyang, Clemson University

Our main goal in this talk is to show that one can skip gradient computation for gradient descent type methods applied to certain structured convex programming (CP) problems. To this end, we first present an accelerated gradient sliding (AGS) method for minimizing the summation of two smooth convex functions with different Lipschitz constants. We show that the AGS method can skip the gradient computation for one of these smooth components without slowing down the overall optimal rate of convergence. This result is much sharper than the classic black-box CP complexity results especially when the difference between the two Lipschitz constants associated with these components is large. We then consider an important class of bilinear saddle point problem whose objective function is given by the summation of a smooth component and a nonsmooth one with a bilinear saddle point structure. Using the aforementioned AGS method for smooth composite optimization and smoothing technique, we show that one only needs  $\mathcal{O}(1/\sqrt{\epsilon})$  gradient computations for the smooth component while still preserving the optimal  $\mathcal{O}(1/\epsilon)$  overall iteration complexity for solving these saddle point problems. We demonstrate that even more significant savings on gradient computations can be obtained for strongly convex smooth and bilinear saddle point problems.

# **An Adaptive Approach for Stochastic Partial Differential Equations using Local Sample-Based Approximations**

Wilkins Aquino, Duke University

We present in this work an adaptive approach for solving partial differential equations (PDE) with random inputs based on local, reduced bases approximations. The main goal of this research is to develop an accurate and scalable approach to solve SPDEs. To this end, we have developed a strategy based on two building blocks: 1) constructing efficient and accurate local approximations over Voronoi cells, and 2) an adaptive strategy for constructing the Voronoi partitioning of the sample space. The first pillar of our approach is a local basis defined over each Voronoi-cell constructed from local solutions at a fixed number of the closest samples to the center of the cell as well as the gradient at the center of the cell. The second ingredient of our approach is an adaptive approach to sequentially select the Voronoi-cell centers. To this end, we adopt a greedy sampling using a rigorous a-posteriori error estimator, which in turn is based on an efficient approximation of the stability constant of the discrete operator. We demonstrate the performance of our proposed approach through several numerical examples with up to thirty (30) stochastic dimensions. When compared to existing methods such as Monte Carlo and Adaptive Sparse Grid, our results indicate that our proposed approach can significantly reduce computational cost for the same accuracy.

# DC Formulations and Algorithms for Sparse Optimization Problems

Jun-ya Gotoh, Chuo University, Japan

This is a joint work with Akiko Takeda and Katsuya Tono

In this paper a DC (Difference of two Convex functions) formulation approach for sparse optimization problems is proposed. First we provide an exact DC representation of the cardinality constraint by using the largest-k norm. Next we show exact penalties for quadratic minimization problems which often appear in practice. A DC Algorithm (DCA) is presented, where the dual step at each iteration can be efficiently carried out due to the accessible subgradient of the largest-k norm. Furthermore, we can solve each DCA subproblem in linear time via a soft thresholding operation if there are no additional constraints. The framework is extended to the rank-constrained problem as well as the cardinality and the rank-minimization problems. Numerical experiments demonstrate the efficiency of the proposed DCA in comparison with existing methods which have other penalty terms.

## On the dynamic consistency of hierarchical risk-averse decision problems

Getachew K. Befekadu, NRC/AFRL & Department of Industrial and Systems

Engineering, University of Florida - REEF

Eduardo L. Pasiliao, Munitions Directorate, Air Force Research Laboratory

In this talk, we consider a risk-averse decision problem for controlled-diffusion processes, with dynamic risk measures, in which there are two risk-averse decision makers (i.e., *leader* and *follower*) with different risk-averse related responsibilities and information. Moreover, we assume that there are two objectives that these decision makers are expected to achieve. That is, the first objective being of *stochastic controllability* type that describes an acceptable risk-exposure set vis-à-vis some uncertain future payoff, and while the *second one* is making sure the solution of a certain risk-related system equation has to stay always above a given continuous stochastic process, namely *obstacle*. In particular, we introduce multi-structure, time-consistent, dynamic risk measures induced from conditional  $g$ -expectations, where the latter are associated with the generator functionals of two backward-SDEs that implicitly take into account the above two objectives along with the given continuous obstacle process. Moreover, under certain conditions, we establish the existence of optimal hierarchical risk-averse solutions, in the sense of viscosity solutions, to the associated risk-averse dynamic programming equations that formalize the way in which both the *leader* and *follower* consistently choose their respective risk-averse decisions. Finally, we remark on the implication of our result in assessing the influence of the *leader's* decisions on the risk-averseness of the *follower* in relation to the direction of *leader-follower* information flow.

Keywords: Dynamic programming equation, forward-backward SDEs, hierarchical risk-averse decisions, value functions, viscosity solutions.

## **Risk Averse Weapon-Target Assignment Problems**

Konstantine Pavlikov, University of Florida

The classical Weapon-Target Assignment (WTA) problem seeks to find an optimal assignment of weapons to targets that maximizes the expected (weighted) number of destroyed targets. This study recognizes that there can be two types of uncertainty about a missile of a certain weapon type to fail to destroy a target: (i) a missile itself can be destroyed on its way to the target and (ii) given that a missile reaches the target, it may fail due to other reasons, e.g., missile malfunction, target protection, and so on. The first objective of this study is to separate these uncertainties, introduce specific assumptions on each of them, and incorporate them back into a joint model. The second objective of this study is to employ recently developed risk averse generalizations of the maximum expected covering location problem in order to create a wider class of risk averse WTA models that enjoy solution algorithms of polynomial complexity.

## **Futures Hedge with the Maximum Loss Control by Buffered Probability of Exceedance and Conditional Value-at-Risk**

Xiao-Lin Suna, School of Transport, Shanghai Maritime University, Shanghai, China

The big loss risk aversion has important implications for hedging. In particular, in contrast with the unambiguous effect that minimum-variance hedging has on the standard deviation, it can actually increase the negative skewness and kurtosis of hedge portfolio returns. Thus, the reduction of probability of big loss events can be significantly lower than the reduction in standard deviation. In this study, we provide Buffered Probability of Exceedance (bPOE) and Conditional Value-at-Risk (CVaR) to control the probability of big loss. We minimize bPOE and CVaR of oil futures hedging portfolio, which has important practical relevance both consistently and significantly. Our findings are robust to different estimation windows, hedging horizons and can be partly explained by the effect of estimation error and model misspecification.

Keywords: Maximum loss; variance change; minimum variance hedging; buffered Probability of Exceedance; Conditional Value-at-Risk.

# On Expectation-Type Risk Quadrangles and Smoothed Buffered Probability Of Exceedance

Alexander Mafusalov, University of Florida

An important class of risk and deviation measures is generated by expectation type measures of error and regret. A class of sublinear error measures is considered. For this class, it is shown that error measure is obtained as a composition of kernel density estimation and Koenker-Bassett error. Furthermore, corresponding statistic is a quantile of the density estimate, and corresponding risk measure is CVaR of the density estimate. Buffered probability of exceedance (bPOE) is a risk-averse alternative to probability of exceedance and cumulative distribution function. Minimization of bPOE is reduced to convex programming or even LP for a wide class of problems. However, being a non-smooth function, bPOE is not always well suited for gradient optimization. In addition, bPOE reverses the curve of CVaR values, while another family of risk measures might be preferable in a given application. This paper introduces a new class of smooth probability-like uncertainty measures, obtained as bPOE of the density estimate. Dual representations and other mathematical properties, along with advantages in optimization, are studied.

## Mitigating Information Asymmetry in the Liver Allocation System

Sepehr Nemati Proon, University of Florida

Zeynep G. Icten, University of Pittsburgh

Lisa M. Maillart, University of Pittsburgh

Andrew J. Schaefer, Rice University

In accordance with the National Organ Transplant Act (NOTA), which requires the efficient and equitable allocation of donated organs, the United Network for Organ Sharing (UNOS) prioritizes patients on the liver-transplant waiting list within given geographic areas based mainly on their most recently reported health status. Accordingly, UNOS requires patients to update their health status at a frequency that depends on their last reported health status. However, patients may elect to update any time within the required timeframe, which creates opportunities to game the system, leading to information asymmetries between UNOS and the patients on the waiting list. This information asymmetry can be alleviated through more frequent updating requirements, but at the price of an increased update burden (e.g., data collection costs and patient inconvenience). We propose a model that determines health reporting requirements that simultaneously minimize these two (possibly conflicting) criteria, i.e., inequity due to information asymmetry and update burden. Calibrating the model with clinical data, we examine (i) the degree to which an individual patient can benefit from the flexibility inherent to the current health reporting requirements, and (ii) alternative recommendations that dominate the current requirements with respect to the two criteria of interest.

# **Error Estimates in Optimization and Application to Risk-Averse Set-Based Design**

Johannes Royset, Naval Postgraduate School

Approximation is central to many optimization problems and the supporting theory provides insight as well as foundation for algorithms. In this presentation, we lay out a broad framework for quantifying approximations by viewing finite- and infinite-dimensional constrained minimization problems as instances of extended real-valued lower semicontinuous functions defined on a general metric space. Since the Attouch-Wets distance between such functions quantifies epi-convergence, we are able to obtain estimates of optimal solutions and optimal values through estimates of that distance. We also consider the stability of near-optimal solutions and level-sets, and the application to risk-averse set-based design of physical systems.

## **Data-driven appointment scheduling under uncertainty**

Petar Momcilovic, University of Florida

Joint work with Avi Mandelbaum (Technion) and Nikos Trichakis (MIT)

We develop a novel, data-driven approach to deal with appointment sequencing and scheduling in a multi-server system, where both customer punctuality and service times are stochastic. We calibrate our model using a data set of unprecedented resolution, gathered at a large-scale outpatient oncology practice, and illustrate how our approach can be utilized to improve infusion scheduling. We also demonstrate the performance of our approach by comparing it with existing state-of-the-art sequencing and scheduling algorithms.

**Nonparametric Estimation of Distribution and Various Characteristics:  
Quantiles (VaR), Superquantiles (CVaR), Probability of Exceedance (POE),  
Buffered Probability of Exceedance (bPOE)**

Stan Uryasev, University of Florida

Alexander Mafuslalo, University of Florida

Georgi Pertaia, University of Florida

Grigori Zrazhevsky, National University of Kiev, Ukraine

Alexander Golodnikov, Institute of Cybernetics, Kiev, Ukraine

The paper presents an approach for nonparametric estimation of the distribution of a random value. The setting is as follows: 1) in-sample dataset (standard design matrix for linear regression) is available; 2) a new observation of factors is obtained; 3) the new observation of factors and the in-sample dataset are used for building the conditional estimate of the distribution. We introduced a new Q3 class of distributions with positive third derivative of quantile function. Q3 class of distributions belongs to a class of unimodal distributions (such as Weibull distribution). Quantile regression was done for multiple confidence levels in “one shot”. Additional constraints were imposed to 1) assure monotonicity of the quantiles for difference confidence levels; 2) Q3 constraints assuring unimodality. The estimated conditional distribution was used for calculation of several characteristics (under condition that a new observation of factors is available): Quantiles (VaR), Superquantiles (CVaR), Probability of Exceedance (POE) for different thresholds, Buffered Probability of Exceedance (bPOE) for different thresholds. CVaR estimates obtained from conditional distributions were compared with the direct Superquantile (CVaR) regression estimates. The estimates showed quite good correspondence.

## Apex - the software for the oil pipeline defect detection

Oleg Diyankov, Neurok Software LLC, 515 E. Crossville road, Roswell, GA, 30075

The talk presents algorithms and the software code Apex for oil pipeline faults/defects detection, which have been developed by the team of up to 20 mathematicians, engineers and software developers. The code is written in C++ and includes about 250,000 lines. One of the important functions, implemented in the code, is 3D modeling of the magnetic field distribution:

$$\operatorname{div}(\beta) = 0, \beta = \mu(|H|)H, H = \nabla\psi$$

here  $\operatorname{div} = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$ ,  $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$  and  $\psi = \psi(x, y, z)$  is the magnetic potential. For estimation of the level of danger of defects we used various unique filters for the real data analysis and an artificial neural network for predicting the size of defects (length, depth and width). Using this code allows predicting the class of danger of defects with very high accuracy. This software is now used by a Russian state-owned transport monopoly Transneft (the largest oil pipeline company in the world), which operates more than 40,000 kilometers of pipelines.

# **Robust Buffered Probability Minimization and Application to Support Vector Machines**

Matthew Norton, University of Florida

In this presentation, we show that a multitude of support vector machine formulations can be posed as Robust Buffered Probability minimization problems. This reformulation reveals a new connection between regularization strategies and robustness, or uncertainty, assumptions. Furthermore, this formulation reveals that non-convex formulations can be viewed as optimistic robust optimization problems, while convex formulations can be viewed as pessimistic. Finally, we show that a mixture of optimism and pessimism provides new insight into non-convex regularization strategies, particularly of the sparsity inducing variety.