

Workshop on Risk Management Approaches in  
Engineering Applications  
Gainesville, Florida-USA, November 9th-10th, 2015

Organizers: Matthew Norton, Aleksandr Mafusalov, Stan Uryasev

## **Statistical inference of empirical estimates of stochastic programs**

Alexander Shapiro, School of Industrial and Systems Engineering, Georgia Institute of Technology, ashapiro@isye.gatech.edu

In this talk we discuss statistical properties of empirical (sample) estimates of the optimal value and optimal solutions of stochastic programs. Examples include the classical Maximum Likelihood method, Sample Average Approximation (SAA) approach to two and multistage stochastic programming and risk averse stochastic optimization. We discuss consistency (Law of Large Numbers), asymptotics (Central Limit Theorem) and Large Deviations type results.

## **Optimization problems with stochastic order constraints**

Darinka Dentcheva, Department of Mathematical Sciences, Stevens Institute of Technology, darinka.dentcheva@stevens.edu

Stochastic orders formalize preferences among random outcomes and are widely used in statistics and economics. We analyze stochastic optimization problems involving stochastic-order relations as constraints that relate performance functionals, depending on our decisions, to benchmark random outcomes. We discuss the relation of univariate and multivariate stochastic orders to utility functions, conditional value at risk, and to coherent measures of risk. The model provides a link between various approaches for risk-averse optimization. We indicate numerical approaches to the problems and outline several applications.

## Risk-Averse Control of Markov Systems

Andrzej Ruszczynski, Department of Management Science and Information Systems,  
Rutgers University, rusz@business.rutgers.edu

We shall consider the challenges of modeling risk in dynamical systems, in particular, controlled Markov systems. We shall refine the concept of time consistency of risk measures for such systems, introducing conditional stochastic time consistency. We shall also introduce the concept of Markovian risk measures and derive their structure. This will allow us to derive a risk-averse counterpart of dynamic programming equations. We shall discuss solution methods and present illustrative examples. Finally, we shall mention extensions to partially observable models and continuous-time models.

## **Tensor methods: A tool for high-dimensional problems**

Ivan Oseledets, Skolkovo Institute of Science and Technology, Moscow, Russia,  
ivan.oseledets@gmail.com

High-dimensional problems arise in a variety of applications ranging from physics to data mining. There are many financial applications that can be formulated as high-dimensional problems: stochastic partial differential equations, path integrals, integrals over Brownian motion. In this talk I will first present a brief introduction to tensor decompositions of multivariate functions and numerical methods for the efficient computation of such decompositions. In the second part of the talk I will present an application of these tools to the computation of path integrals arising in the evaluation of financial instruments and compare the result with Monte-Carlo methods. This talk is based on joint work with M. Litsarev, “Low-rank approach to the computation of path integrals.”

## **Robust empirical optimization is almost the same as mean-variance optimization**

Jun-ya Gotoh, Department of Industrial and Systems Engineering, Chuo University,  
jgoto@indsys.chuo-u.ac.jp  
Mike J. Kim  
Andrew E.B. Lim.

We formulate a distributionally robust optimization problem where the empirical distribution plays the role of the nominal model and the decision maker optimizes against a worst-case alternative, where the deviation of the alternative distribution from the nominal is controlled by a divergence penalty in the objective. Our main finding is that robust empirical optimization is essentially equivalent to solving an in-sample mean-variance problem, which provides insight into the mechanism by which distributionally robust empirical optimization achieves its robustness. Specifically, the equivalence shows that robustness is achieved by controlling the in-sample variability of the reward distribution, i.e., selecting decisions with low variability in the reward. We consider two applications, the empirical newsvendor problem and the empirical portfolio optimization problem. Our numerical experiments show that the primary benefit of using robust empirical optimization is its ability to produce solutions with low out-of-sample variability in the reward, which is consistent with our main theoretical finding. In the case of the portfolio choice problem, we draw on the insights from our main result to introduce a robust version of cross-validation that is useful in applications where distributions from resampling are sensitive to data variability and model misspecification.

## **Simulating future test and redesign considering epistemic model uncertainty**

Nathaniel Price, University of Florida, NatBPrice@gmail.com

At the initial design stage engineers often rely on low fidelity models that have high epistemic uncertainty. Traditional safety-margin-based deterministic design has used testing to reduce epistemic uncertainty and achieve high levels of safety. Testing is used to calibrate models and prescribe redesign when tests are not passed. After calibration, reduced epistemic model uncertainty can be leveraged through redesign to restore safety or improve design performance; however, redesign may be associated with substantial costs or delays. In this study, a methodology is described for optimizing the safety margin based design, testing, and redesign process to allow the designer to tradeoff between the risk of future redesign and the possible performance and reliability benefits. The proposed methodology based on using a Kriging surrogate to represent the epistemic model uncertainty is applicable in a wide range of design problems. The method is illustrated on a cantilever beam design problem where there is mixed epistemic model error and aleatory parameter uncertainty.

## **Optimization of Value-at-Risk: Computational Aspects of MIP Formulations**

Konstantin Pavlikov, Air Force Research Laboratory, University of Florida,  
kpavlikov@ufl.edu

Optimization of Value-at-Risk is an important problem both from theoretical and practical standpoints. It can be represented through a class of chance-constrained optimization problems, which are generally hard to solve. Mixed integer linear problem formulations with big M constants is a standard way to approach such problems, where tightness of these constants plays an important role in a solver's performance. This study aims to improve the tightness of existing big Ms by explicitly incorporating the bounds on the optimal solution into the problem formulation. Moreover, the lower bound is demonstrated to play the key role in obtaining tight big M constants and a procedure to tighten this bound is discussed. Finally, a "two-stage" solution approach is proposed, where the first stage solely deals with tightening the bounds, and the second stage solves the problem to optimality. Numerical experiments suggest that proposed solution methods can decrease solution time by up to 80% compared to the most recent benchmark. In addition, our solution techniques help to reduce the memory used by the solver, which may allow to handle larger problem instances using the same hardware.

## **Analysis of Tropical Storm Damage using Buffered Probability of Exceedance**

Justin R. Davis, Department of Civil and Coastal Engineering,  
justin.r.davis@essie.ufl.edu

Relying solely on probability of exceedance (POE) or in combination with other statistical measures such as mean or standard deviation does not always give a clear picture of data which have heavy tailed distributions. In this paper, the new concept of buffered probability of exceedance (bPOE) is explained and reduced to a simple algorithm. Through averaging of data in the tail, bPOE compactly presents both probability as well as density of heavy tailed distributions and has the capability to revolutionize the concept of risk-averse engineering. Once defined, bPOE is then demonstrated through its application to normalized tropical storm damage along the Atlantic and Gulf of Mexico coasts of the U. S. (average damage \$6.961 billion). For instance, with a \$50 billion threshold, the probability of exceedance (POE) is 3.7% while the bPOE is significantly higher at 9.7%. It can be shown, under general assumptions, that bPOE is more than double POE. For a 25% probability, the value-at-risk (quantile) determined using POE is \$4.785 billion, while the conditional-value-at-risk (average of damage in excess of the quantile) determined by the bPOE is \$25.176 billion. For a fixed probability, a general formula has been developed for a quantity called ?expected excess?, the difference between the damage associated with POE and bPOE multiplied by the bPOE. Formally thought of as the damage above the value-at-risk, it is, for example, the minimum amount an insurer would need to charge in premiums to ?break even?. For example, the \$20.391 difference shown for the 25% probability level, leads to an expected excess of  $0.25 * \$20.391 = \$5.098$  billion. In addition to applying bPOE to an entire dataset, it can also be applied to statistically meaningful subsets. For example, subdividing the data by landfall state, at the 50% probability level, Florida, the state most often hit by tropical storms also has the highest value for expected excess at \$8.111 billion; thus, quantifying the need for insurers to charge Floridians high insurance premiums.

## **Efficient solution of forward stochastic equations for inverse problems**

Mircea Grigoriu, Department of Civil and Environmental Engineering, Cornell  
University, mdg12@cornell.edu

Inverse problems deliver properties of uncertain parameters in the definition of input-output relationships from output observations. They can be formulated as constrained optimization problems or Bayesian inference problems. Both formulations require efficient algorithms for solving forward problems. Our focus in this presentation is on the development of an efficient algorithm for solving forward problems in a stochastic setting, e.g., partial differential equations (PDEs) with random coefficients, referred to as stochastic partial differential equations (SPDEs). The algorithm involves three steps. First, parametric models are constructed for the random fields in the definition of stochastic equations, i.e., deterministic functions of space and/or time arguments that depend on finite numbers of random variables collected in vectors  $Z$ . Then, solutions  $U(x)$  of, e.g., SPDEs become parametric random fields denoted by  $U(x, Z)$ . Second, stochastic reduced order models (SROMs), i.e., random vectors  $\bar{Z}$  with finite numbers of samples, are developed for  $Z$  such that the probability laws of  $Z$  and  $\bar{Z}$  are similar in some sense. Third, surrogate models  $\bar{U}(x, Z)$  are constructed for  $U(x, Z)$  based on SROMs  $\bar{Z}$ . The surrogates are piecewise linear approximations of the mappings  $Z \rightarrow U(x, Z)$  indexed by  $x$ . The proposed algorithm is applied to calculate the apparent conductivity of a material specimen whose conductivity fluctuates randomly in space.

## **Monotone investment performance measures based on reward-to-variability ratios**

Mikhail Zhitlukhin, Steklov Mathematical Institute, mikhailzh@mi.ras.ru

We develop a class of investment performance measures defined through the ratio of investment's reward and variability, which are quantified by coherent utility and deviation measures. The idea goes back to the famous Sharpe ratio, which is the ratio of the mean and the standard deviation of a strategy's returns. However, one major problem of the Sharpe ratio is that it is not monotone: an investment strategy with higher return may have a lower Sharpe ratio. We will show how this problem can be resolved by introducing a "monotonization" procedure. It will also allow to construct new performance measures by taking general coherent utility and deviation measures. We will apply the new performance measures to a database of investment funds' returns and analyze the difference with results produced by classical approaches.

## **On the attainable distributions of diffusion processes pertaining to a chain of distributed systems**

Getachew K. Befekadu, Department of Mechanical and Aerospace Engineering,  
University of Florida, gbefekadu@ufl.edu

Eduardo L. Pasiliao, Air Force Research Laboratory, elpasiliao@gmail.com

In this talk, we consider a diffusion process pertaining to a chain of distributed systems with random perturbations that satisfies a weak Hormander type condition. In particular, we consider the following stochastic control problem with two objectives. The first one being of a reachability-type that consists of determining the set of attainable distribution laws at a given time starting from an initial distribution law; while the second one involves minimizing the relative entropy of the attainable distribution law with respect to the initial law. Using the logarithmic transformations approach introduced by Fleming, we provide a sufficient condition on the existence of an optimal admissible control for such a stochastic control problem which is amounted to changing the drift term by a certain perturbation suggested by Jamison in the context of reciprocal processes. Moreover, such a perturbation coincides with a minimum energy control among all admissible controls forcing the diffusion process to the desired attainable distribution law starting from the initial law. Finally, using measure transform techniques, we characterize the most probable path-space for the diffusion process corresponding to such changes in the drift term of the distributed systems.

## **Soft Margin Support Vector Classification as Buffered Probability Minimization**

Matthew Norton, University of Florida, mdnorton@ufl.edu

In our research, we provide theoretical insights into the nature of the popular C-SVM, soft-margin support vector classifier. We prove that the C-SVM, formulated with any regularization norm, is equivalent to minimization of Buffered Probability of Exceedance (bPOE). This equivalence allows us to gain new insights into the C-SVM. For example, along with insights inspired by bPOE, we show that the choice of regularization norm implies a distance metric for ‘margin maximization.’ To help provide such theoretical insights, we introduce a new SVM formulation, the Extended Soft Margin Support Vector Machine (EC-SVM), named in this way to emphasize its relation to the Extended v-Support Vector Machine (Ev-SVM). We derive the EC-SVM as a bPOE minimization problem and show that the C-SVM is equivalent to the EC-SVM. Throughout, we utilize bPOE and the EC-SVM to interpret the C-SVM in a new, surprising way. We also connect the EC-SVM with the Ev-SVM, showing that they produce the same set of optimal solutions, helping to fully connect soft margin support vector classification with superquantile and bPOE concepts.

## **Buffered Probability of Exceedance: Methodology and Software**

Stan Uryasev, Risk Management and Financial Engineering Laboratory, University of Florida, uryasev@ufl.edu

This paper investigates a new probabilistic characteristic called buffered probability of exceedance (bPOE). bPOE counts tail outcomes averaging to some specific threshold value. Minimization of bPOE can be reduced to Convex and Linear Programming. We will explain the approach with a dynamic stochastic optimization problem. At every time moment, a supplier wants to supply some predetermined amount of product. Production costs are stochastic and produced amount of product is described by a linear dynamic equation with uncertain coefficients. We illustrate this general model with an application in finance, which is called Cash Matching of a Bond Portfolio. We minimize bPOE that liabilities exceed assets. We discuss both methodological and software issues for the considered problem.

## Multidimensional Probability of Exceedance: Dual Representations and Multivariate Stochastic Dominance

Aleksandr Mafusalov, University of Florida, mafusalov@ufl.edu

Probability of exceedance (POE) is widely used but has major drawbacks. Buffered probability of exceedance (bPOE) is a conservative approximation of POE and eliminates some of its drawbacks. We suggest a new multidimensional generalization of bPOE (M-bPOE). M-bPOE is used to control exceedances for several random variables simultaneously. Calculation formulas and optimization formulations for M-bPOE are provided. Dual representations of bPOE and M-bPOE are studied. Stochastic order based on M-bPOE generalizes second order (SSD) and convex stochastic dominance for univariate case. For multivariate case, M-bPOE order unifies convex-linear SSD and lift zonoid dominance. Distribution approximation problems with M-bPOE constraints are studied. M-bPOE dominance over discrete distribution is expressed with a finite number of linear constraints. Entropy maximization with M-bPOE constraints and variance constraints leads to the maximum-of-Gaussians form of the optimal solution.

## **Application of VaR and CVaR risk measures in the Financial Time Series Analysis**

Nataly Zrazhevska, National Technical University of Ukraine,  
natasha.zrazhevska@gmail.com

We investigated methods for calculation of VaR and CVaR risk measures in financial applications. We used GARCH model for calculating and analyzing dynamical VaR and CVaR for time series with strong volatility. Innovations of GARCH model have heavy tails. This paper uses main ideas of EVT and POT methods.

## **Cash flow matching with risks controlled by bPOE and CVaR**

Viktor Kuzmenko, Danjue Shang, and Stan Uryasev

Bond immunization is an important topic in portfolio management. This paper demonstrates a scenario based optimization framework for solving a cash flow matching problem where the time horizon of the liabilities is longer than the maturities of available bonds and the interest rates are uncertain. Bond purchase decisions are made each period to generate cash flow for covering the obligations in future. Cash flows depend upon uncertain future prices of bonds. We use Buffered Probability of Exceedance (bPOE) and Conditional Value-at-Risk (CVaR) to control risk of shortfalls. The initial cost of the hedging portfolio of bonds is minimized and optimal positions of bonds are calculated at all time periods. We used Gurobi and Portfolio Safeguard (PSG) optimization packages to solve optimization problems.

## **Linear Regression and Applications of MIP for Optimal Selection of Variables**

Filiz Ersoz, Karabuk University, Turkey, fersoz@karabuk.edu.tr

Stan Uryasev, University of Florida, USA, uryasev@ufl.edu

We present comparison of several approaches for selecting the best subset of explanatory variables in a multiple linear regression model. Adjusted R<sup>2</sup> criterion is used to evaluate subsets of selected variables. We conducted a case study for datasets downloaded from the UCI Machine Learning Repository of the University of California. 10 different datasets with different sample sizes and number of factors are included in the analysis. We compared our calculations with the optimal results presented in Myashiro and Takano, ?Mixed Integer Second-Order Cone Programming Formulations for Variable Selection?. We have run several commercial packages, IBM SPSS Statistics, IBM SPSS Modeler (Data Mining), Minitab and Portfolio Safeguard to benchmark the known optimal subsets of variables. Our calculations show that professional packages usually deliver sub-optimal adjusted R<sup>2</sup> subsets of variables.

## An optimal randomized incremental gradient method

Guanghui Lan, University of Florida, Department of Industrial and Systems  
Engineering, glan@ise.ufl.edu

We consider a class of finite-sum convex optimization problems whose objective function is given by the summation of  $m$  ( $\geq 1$ ) smooth components together with some other relatively simple terms. We first introduce a deterministic primal-dual gradient (PDG) method that can achieve the optimal black-box iteration complexity for solving these composite optimization problems using a primal-dual termination criterion. Our major contribution is to develop a randomized primal-dual gradient (RPDG) method, which needs to compute the gradient of only one randomly selected smooth component at each iteration, but can possibly achieve better complexity than PDG in terms of the total number of gradient evaluations. More specifically, we show that the total number of gradient evaluations performed by RPDG can be  $\mathcal{O}(\sqrt{m})$  times smaller, both in expectation and with high probability, than those performed by deterministic optimal first-order methods. We also show that the complexity of the RPDG method is not improvable by developing a new lower complexity bound for a general class of randomized methods for solving large-scale finite-sum convex optimization problems. Moreover, through the development of PDG and RPDG, we introduce a novel game-theoretic interpretation for these optimal methods for convex optimization.

## **OPTIMUS Software**

Oleg Diyankov, Anastasia Lomokhova, Andrey Shumkov, Vladislav Pravilnikov, Neurok Software LLC, Alpharetta, GA, oleg.diyankov@neuroksoftware.com

The presentation describes the software intended for the solution of linear programming (LP) problems. The set of algorithms implemented in the code includes the dual simplex method, interior point method and some algorithms for data preprocessing. The software includes a specific preprocessing algorithm which allows accelerating of the solution of so-called Financial Transmission Rights (FTR) problems. The use of this preprocessing algorithm gives from 3 to 5 times acceleration with respect to the CPLEX dual simplex method. The second part of the presentation describes a brand new method for the solution of LP problems based on the solution of ODE. This method is close in some sense to the interior point method; its advantage is that one does not need to seek for a feasible point. The presentation contains some results of numerical experiments.

## **Electricity Market Design with Intermittent Renewable Generation**

Alfredo Garcia, University of Florida

agarcia@ise.ufl.edu

We consider the problem of designing the rules of interaction (pricing, clearing) for power producers and consumers in an electricity market with a significant amount of highly intermittent generation. Recently, several electricity markets have adopted designs in which the market maker identifies do-not-exceed (DNE) limits, i.e the maximum output (for each renewable plant) that the power system can accommodate without sacrificing system reliability. However, the introduction of such rule in combination with existing pricing rules (e.g. locational marginal pricing) may induce an efficient utilization of resources. We consider a market design (i.e. rules for pricing and clearing) that is aimed at identifying the socially optimal dispatch of resources while providing incentives for truthful bidding by market participants.