CASE STUDY: Supply Chain Two-Stage Planning Problem (recourse, avg, linear)

Background

The case study solves an instance of supply chain design problem provided for testing purposes by Peter Schütz (Schütz 2011). The importance of supply chain design was recognized since 1970s. A supply chain management includes design of, planning for and operation of a network of suppliers, production facilities, warehouses, and distribution centers in order to satisfy customer demand. Strategic decisions regarding the design of supply chains affect the ability to efficiently serve customer demand. The design decisions should therefore consider the effect on operational decisions.

This case study models the supply chain design problem as a sequence of splitting and combining processes. The problem is formulated as a two-stage stochastic program. The first-stage decisions are strategic location decisions, whereas the second stage consists of operational decisions. The objective is to maximize the expected profits over the planning horizon. Another variant of objective is to minimize the sum of investment costs and expected costs of operating the supply chain.

The solved problem instance is a real-life problem from the Norwegian meat industry. The task is to balance supply and demand on a weekly basis, ensuring that the right raw materials are available at the right production facilities in order to satisfy demand. The planning horizon is four weeks. Demand for the first week is known with certainty, whereas planning of production and material flow for the remaining three weeks is based on predicted demand. For more details see Schütz (2011).

Below we give a general formulation of the problem. It is reformulated as minimization one. Solutions of the second stage problem for different scenarios give the values of the stochastic Recourse Function.

The solved stochastic linear programming problem contains 22,676 columns and 11,978 rows for the first stage and 74,398 columns and 37,893 rows for every subproblem of the second stage. The number of scenarios is 75. So in the equivalent LP formulation the problem contains 5,602,526 columns and 2,853,953 rows.

We reported performance of AORDA Portfolio Safeguard (PSG) 64 bit version conducted on PC with 2.83 MHz processor.

References


Notations

\( I \) = number of the first stage decision variables, \( i = 1, \ldots, I \) index of variable;

\( K \) = number of the second stage decision variables, \( k = 1, \ldots, K \) index of variable;

\( \tilde{x} = (x_1, \ldots, x_I) \) = vector of the first stage decision variables;

\( \tilde{y} = (y_1, \ldots, y_K) \) = vector of the second stage decision variables;

\( A \) = matrix of the first stage linear constraints;

\( b, \overline{b} \) = vectors of lower and upper bounds for the first stage linear constraints;

\( x_{ij}, \overline{x}_i \) = lower and upper bounds for the first stage variables, \( i = 1, \ldots, I \);

\( \tilde{c}, \overline{d} \) = cost vectors for the second stage objective;

\( T, W \) = matrices of the second stage linear constraints;

\( y_{ik}, \overline{y}_k \) = lower and upper bounds for the second stage variables, \( k = 1, \ldots, K \);

\( p_j \) = probability of scenario \( j, j = 1, \ldots, J \);
$L(j), \ U(j) =$ vectors of lower and upper bounds for the second stage linear constraints for scenario $j$;

$Q(\tilde{x}, j) =$ recourse function for scenario $j$:

$$Q(\tilde{x}, j) = \tilde{c}\tilde{x} + \min_y \{\tilde{d}y\}, \ j \in \{1, ..., J\}$$

subject to

linear constraints for the second stage

$L(j) \leq T\tilde{x} + W\tilde{y} \leq U(j)$,

bounds on the second stage decision variables

$\underline{y}_k \leq y_k \leq \overline{y}_k, \ k = 1, ..., K$;

$\text{avg}(Q(\tilde{x}, j)) = \sum_{j=1}^{J} p_j Q(\tilde{x}, j) =$ Average of Recourse function over scenarios.

**Optimization Problem**

minimizing Average of Recourse function

$$\min_x \text{avg}(Q(\tilde{x}, j)) \quad (\text{CS.1})$$

subject to

linear constraints on the first stage decision variables

$$b \leq Ax \leq b, \quad (\text{CS.2})$$

bounds on the first stage decision variables

$$\underline{x}_i \leq x_i \leq \overline{x}_i, \ i = 1, ..., l. \quad (\text{CS.3})$$