CASE STUDY: Optimal Crop Production and Insurance Coverage
(pr_pen, var_risk, cvar_risk, cardn_pos)

Background

This case study investigates an optimal crop production and insurance coverage under three types of risk constraints: CVaR, VaR, and Probability Exceeding Penalty constraint.

Farmers face uncertainties of crop yields and market prices affecting their profit. These uncertainties contribute to the profit risk. To hedge the risk, farmers can purchase insurance against the uncertainties of yields and prices. There are several insurance policies on the market providing different protections with different prices. This case study finds the best planting plan and insurance policy creating maximum profit under an acceptable risk exposure.

Crop yields are affected by climate type. Hansen et al. (1998) shows that El Nino Southern Oscillation (ENSO) is a strong driver of seasonal climate variability that impact cotton and peanut crop yields in the southeastern US. Climate can be categorized into three phenomena: El Nino, La Nina and Neutral. The phenomenon can be predicted by the sea surface temperature. When the temperature is normal, the phenomenon is called Neutral. If the temperature is lower or higher than normal, the phenomenon is referred to as La Nina or El Nino, respectively. Each phenomenon has different effects on the crop yields due to the different temperature and rainfall. For instance, El Nino brings more rainfall and cooler temperatures, while La Nina brings less rainfall and warmer temperatures than normal.

The harvest price of crops is an important factor affecting farmers' income. Based on the multivariate time series of historical crop prices, we calculated the variance of prices for each crop and covariance of prices between crops. Scenarios of the prices of crops were generated by multivariate simulation. The procedure followed the methodology of Letson et al. (2005).

There are three main types of crop insurance: the Actual Production History crop insurance (APH), the Crop Revenue Coverage insurance (CRC), and the Catastrophic Insurance Coverage (CAT). APH assures a percentage of the farmers' history yield. If the yield becomes lower than the insured yield, the insurance pays an indemnity covering the difference between the insured yield and the real yield. CRC assures income by indemnifying farmers based on historical average yield and the market price. If the actual yield multiplied by the established price or actual market price is lower than an indemnified income level, a farmer is entitled to an insurance payment. CAT can be defined as an APH policy at 50% yield coverage with 55% price base election.

Several studies have addressed the impacts of the ENSO based climate forecasts on the selection of crop insurance policy. Cabrera et al. (2005) used the utility function to address farmers' risk aversion. Utility function is widely used for theoretical and mathematical purposes. The disadvantage is that farmers can not specify their utility functions. Instead Lui (2005) employed CVaR as the risk measure and formulated the problem as a quadratic problem. This case study improves the quadratic model proposed by Lui. Similarly, we consider a model for planting and insurance by maximizing the total profit under a risk level measured by CVaR, VaR, and Probability Exceeding Penalty.

References

- Cabrera, V.E., Fraisse C.W., Letson D., Posesta G., Novak J.(2005): Impact of climate information on reducing farm risk by optimizing crop insurance strategy. Working paper (2005).
- Hansen, J.W., Hodges, A.W., Jones, J.W. (1998): ENSO influences on agriculture in the Southeastern US. *J Climate* 11, 404-411.
- Letson, D., Podestá, G.P., Messina, C.D., Ferreyra, A. (2005): The uncertain value of perfect ENSO phase forecasts: stochastic agricultural prices and intra-phase climatic variations. *Climatic Change,* 69, 163-196.
- Men, C, Liu, J. (2005): A conditional value-at-risk model for optimizing crop insurance strategy. Working paper (2005).

Notations

$K =$ number of crop types;
$T_k =$ number of planting dates for crop *k;*
$I_k =$ number of insurance policies for crop *k;*
$J =$ number of scenarios*;*
$q_k =$ planting area (in acres) available for crop *k*;
$C_k =$ planting cost per acre of crop *k;*
$R_{ki} =$ premium of insurance policy *i* for crop *k* per acre*;*
$x_{kti} =$ number of acres of land for crop *k* planting on date *t* and insured by policy *i;*
$x_{ki}^a =$ additional variable indicating selection of insurance policy *i* for crop *k;*
$\theta_{kti} =$ (random) revenue of crop *k* per acre planted on date *t* and insured by policy *i;*
$\theta_{ktij} =$ revenue of crop *k* per acre planted on date *t* and insured by policy *i* under scenario *j*;
$L(x,\theta) = -\sum_{k=1}^{K}\sum_{t=1}^{T_k}\sum_{i=1}^{I_k}\theta_{kti}x_{kti} =$ loss function;
$L^* =$ threshold of losses in probability exceeding penalty constraint;
$p =$ the upper bound of the probability exceeding penalty constraint;
$b =$ upper bound on $CVaR_\alpha\left(L(x,\theta)\right)$;
$v =$ upper bound on $VaR_\alpha(L(x,\theta))$;
$y_{ktj} =$ yield of crop *k* per acre planted on date *t* under scenario *j*;
$y_k =$ (historical) average yield of crop *k* per acre;
$P_{kj} =$ market price of crop *k* per pound under scenario *j*;
$P_k =$ price base of crop *k* per pound.

Optimization Problem

Maximizing expected profit

$$\max_{x_{kti}} \sum_{k=1}^{K}\sum_{t=1}^{T_k}\sum_{i=1}^{I_k} E\theta_{kti}x_{kti} \tag{CS1}$$

subject to

planting area constraint for each crop k

$$\sum_{t=1}^{T_k}\sum_{i=1}^{I_k} x_{kti} = q_k, \quad k = 1,\ldots,K \tag{CS2}$$

joint constraint on planting area and insurance policy

$$\sum_{t=1}^{T_k} x_{kti} \le q_k x_{ki}^a, \quad i = 1,\ldots,I_k;\, k = 1,\ldots,K \tag{CS3}$$

each crop k can be insured by at most one policy

$$Cardinality\ Positive\left(x_k^a, w\right) \le 1,\ k = 1,\ldots,K\ , \tag{CS4}$$

risk constraint (CVaR, VaR, and Probability Exceeding Penalty)

$$CVaR_\alpha(\,L(\,x,\theta\,)) \le b \tag{CS5}$$

$$\text{or } VaR_\alpha(L(x,\theta)) \le v \tag{CS6}$$

$$\text{or } \Pr ob(L(x,\theta) \ge L^*) \le p \tag{CS7}$$

constraint on additional variables

$$0 \le x_{ki}^a \le 1, \quad i = 1,\ldots,I_k;\, k = 1,\ldots,K\ , \tag{CS8}$$

lower bounds on variables

$$0 \le x_{kti} \le q_k, i = 1,\ldots,I_k;\ t = 1,\ldots,T_k \tag{CS9}$$

Scenarios Generation

Random revenue θ_{kti} has *J* equally probable scenarios θ_{ktij}, which can be calculated as follows:

$$\theta_{ktij} = y_{ktj}P_{kj} + indemnity - C_k - R_{ki}, \tag{CS10}$$

where

$$indemnity \text{ for APH} = \begin{cases} (y_{ki}^* - y_{ktj})P_k, & if \quad y_{ki}^* > y_{ktj} \\ 0, & if \quad y_{ki}^* \le y_{ktj} \end{cases}, \tag{CS11}$$

$$y_{ki}^* = \text{APH\%} \times y_k, \tag{CS12}$$

$$indemnity \text{ for CAT} = \begin{cases} (y_{ki}^* - y_{ktj})P_k^*, & if \quad y_{ki}^* > y_{ktj} \\ 0, & if \quad y_{ki}^* \le y_{ktj} \end{cases}, \tag{CS13}$$

$$y_{ki}^* = 50\% \times y_k,\ \ P_k^* = 55\% \times P_k \tag{CS14}$$

$$indemnity \text{ for CRC} = \begin{cases} Insured\ Income - Actual\ Income, & if \quad Insured\ Income > Actual\ Income \\ 0, & if \quad Insured\ Income \le Actual\ Income \end{cases}, \tag{CS15}$$

$$Insured\ Income = y_k \times P_k \times CRC\%, \tag{CS16}$$

$$Actual\ Income = y_{ktj}P_{kj}. \tag{CS17}$$

Parameters of the Problem

Number of types of crop = 2 (Cotton and Peanut)

Number of planting area for each crop = 50 acre

Number of Scenarios = 990 (for each climate: El Nino, La Nina and Neutral)

Number of planting dates for cotton = 4

1. 16 Apr
2. 23 Apr
3. 1 May
4. 8 May

Number of planting dates for peanut = 9

1. 16 Apr
2. 23 Apr
3. 1 May
4. 8 May
5. 15 May
6. 22 May
7. 29 May
8. 5 Jun
9. 12 Jun

Number of insurance policy for peanut (and its premium)

1. No insurance (0)
2. CAT ($2/acre)
3. 65%APH ($8.5/acre)
4. 70%APH ($11.2/acre)
5. 75%APH ($16.6/acre)

Number of insurance policy for cotton (and its premium)

1. No insurance (0)

2. CAT ($2/acre)
3. 65%APH ($19.5/acre)
4. 70%APH ($25.5/acre)
5. 75%APH ($38.2/acre)
6. 65%CRC ($24.8/acre)
7. 70%CRC ($32.4/acre)
8. 75%CRC ($47.9/acre)
9. 80%CRC ($74.1/acre)
10. 85%CRC ($116.9/acre)

Production cost for peanut = $436.95/ acre

Production cost for cotton = $463.97/acre

Average yield for peanut = 2999.54 lb/acre

Average yield for cotton = 650.41 lb/acre

Election price for peanut = $0.1785 / lb

Election price for cotton = $0.63 / lb

CVaR constraint:

The confidence level in CVaR for the portfolio, $\alpha = 90\%$

The upper bound on $CVaR_{\alpha}\left(L(x,\theta)\right)$ of losses $w = 6000$

Probability Exceeding Penalty constraint:

The threshold of losses = 600

The upper bound on exceeding probability = 10%

Three ENSO climate phases are considered in the optimal crop production and insurance coverage problems

1. El Nino
2. La Nina
3. Neutral

For each ENSO climate phase, we consider three different risk constraints, namely, CVaR, VaR and Probability Exceeding Penalty (PrPen) constraint.

Therefore, we have a total of nine problems:

1. problem_crop_insurance_el_nino_CVaR ;
2. problem_crop_insurance_neutral_ CVaR;
3. problem_crop_insurance_la_nina_ CVaR;
4. problem_crop_insurance_el_nino_VaR ;
5. problem_crop_insurance_neutral_ VaR;
6. problem_crop_insurance_la_nina_ VaR;
7. problem_crop_insurance_el_nino_PrPen;
8. problem_crop_insurance_neutral_ PrPen;
9. problem_crop_insurance_la_nina_ PrPen.

These nine problems are grouped by risk constraint (CVaR, VaR, and Probability Exceeding Penalty) into 3 problems:

- Problem1: problem_crop_insurance_CVaR;
- Problem2: problem_crop_insurance_VaR;
- Problem3: problem_crop_insurance_ PrPen.

Three ENSO climate phases are considered for each problem by using three Dataset:

1. Dataset1 for El Nino,
2. Dataset2 for La Nina,
3. Dataset3 for Neutral.