ESI6912 : Fundamentals of Mathematical Programming

Fall 2012

1. Catalog Description:

Credits: 3; Introduction to Mathematical Programming, with an emphasis on fundamental mathematical concepts used in optimization, classical optimization theory and applications of optimization in engineering. Focus on convex analysis (convex sets, separation theorems, convex functions), optimality conditions (Fritz-John & Karush-Kuhn-Tucker) and lagrangian duality.

2. Pre-Requisites and Co-Requisites:

Although there is no formal prerequisite for this class, students should have:

- A good mathematical background. In particular, basic concepts in linear algebra and calculus will be used frequently.
- An ability to prove basic mathematical statements.
- An ability to write simple codes with Matlab or C (or the motivation to learn.)

3. Course Objective:

In this class, we will cover classical models, tools and results in Mathematical Programming. At the end of the class, we expect students to be able to (1) determine when problems they face can be formulated as nonlinear optimization problems, (2) recognize when these problems are convex, (3) know classical results characterizing optimal solutions to these problems and (4) understand how to use these classical results on application problems. Through the course of the class, we also expect that students will develop an ability to formulate mathematical statements precisely and to prove them rigorously . Finally, we expect students to gain some familiarity with commercial algebraic modeling packages such as GAMS.

4. Instructor:

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5. Teaching Assistant:

Name: Danial Davarnia
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Office Hours: By appointment

6. Meeting Times:

Second and third periods, 8:30-10:25am

7. Class/laboratory schedule:

The class meets twice a week (Monday and Wednesday).

8. Meeting Location:

Location: MAEB 0234

9. Material and Supply Fees:

NA.

10. Textbooks and Software Required:

Title: *Nonlinear Programming: Theory and Algorithms*
Author: *Bazaraa, Sherali and Shetty*
Publication date and edition: *Wiley, 2006*, 3rd Edition
ISBN-10 number: *0-471-48600-0*

11. Recommended Reading:

- Bertsekas, *Nonlinear Programming*, Athena Scientific, 1995.
- Luenberger, *Introduction to Linear and Nonlinear Programming*, Addison-Wesley, 1984.

12. Course Outline:

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Week 1: Introduction: Engineering Applications of Optimization
Basic Forms of Optimization Problems, Optimal Solutions; Traffic Equilibria, Curve Fitting & Newsvendor Problems

Week 2: Introduction: Engineering Applications of Optimization/ Modeling with GAMS
Facility Location with Risk Pooling, Portfolio Optimization, Oil Pooling & Mechanical Equilibria; GAMS Modeling

Week 3: Mathematics of Optimization: Algebra Review
Vector Spaces, Distances, Norms, Angles, Linear Maps, Eigenvalues, Matrix Diagonalization Theorems

Week 4: Mathematics of Optimization: Analysis Review

Sequences, Convergence, Open/Closed Sets, Bolzano-Weierstrass Theorem, Continuity, Weierstrass Theorem

Week 5: Convex Analysis: Convex Sets & Projection

Definitions, Examples, Operations Preserving Convexity, Convex Cones, Carathéodory's Theorem, Projection Theorem

Week 6: Convex Analysis: Separation Theorems & Inner Representation of Convex Sets

Separation, Supporting Hyperplanes, Farkas/Gordan Lemmas, Extreme Points and Directions, Minkowski-Weil Theorem, Dantzig-Wolfe Reformulation

Week 7: Convex Analysis: Convex Functions

Definitions, Examples, Epigraphs, Level Sets, Existence of Subgradients, Gradient Conditions, Hessian Conditions

Week 8: Convex Optimization: Convex Programs & Engineering Applications

Definition, Properties, Families of Convex Programs, Examples, Engineering Applications, Convex Relaxations

Week 9: Optimality Conditions: Convex Programs and Unconstrained Optimization Models

Optimality Conditions in Convex Programs, First & Second Order Conditions for Unconstrained Optimization

Week 10: Optimality Conditions: Constrained Optimization Models

Improving & Feasible Directions, First & Second Order Fritz-John Conditions, Examples and Applications

Week 11: Optimality Conditions: Constrained Optimization Models

First & Second Order KKT Conditions, Constraint Qualifications (LIG, Abadie, Slater), Examples and Applications

Week 12: Lagrangian Duality: Lagrangian Functions & Properties

Definitions, Concavity of Lagrangian Dual Function, Subgradients, Subdifferentials, Examples (LP and quadratic duals)

Week 13: Lagrangian Duality: Weak and Strong Duality

Weak duality, Strong duality, Role of Convexity, Saddle Points Conditions, Applications (Conic Duality and IP Lagrangian duals), Relations to KKT, Sensitivity Analysis

Week 14: Lagrangian Duality: Solution methods

Cutting planes, Ellipsoid and Subgradient Algorithms

Week 15: Review

This list of topics and the associated schedule are only tentative. Some topics may be added, some topics might be removed depending on the interest of students and the pace of the class. I encourage you to communicate to me other topics that you would like to see covered in the class. I will do the best I can to accommodate relevant requests.

13. Attendance and Expectations:

Students are expected to attend class and to notify the instructor when they are not able to. Repeated unexcused absences might result in a penalty of up to 10% of the class grade.

14. Grading - Methods of Evaluation:

Homework will be assigned during the course of the semester (four to five in total). Some problems will be simple applications of the material covered in class, some problems will require the proof of theorems and some problems will require computer implementations. Most students will find homework to be challenging and time-consuming.

Students are required to take a midterm, in-class, written exam towards the middle of the semester. The exact date of the exam will be communicated later.

Students are required to take a final exam. The goal is to test the general knowledge and understanding of the class material. The final exam is cumulative. The final covers material described during the class itself but also material studied in the homework and project.
Class grades will be based on: homework average grade (35%), project grade (25%), and final exam grade (40%).

15. Grading Scale:

The (tenative) grading scale for the class is: A (95-100), A- (90-94), B+ (87-89), B (83-86), B- (80-82), C+ (77-79), C (73-76), C- (70-72), D+ (67-69), D (63-66), D- (60-62), F (0-59). Curving might occur in any assignment/exam if the average result is too low.

In order to graduate, graduate students must have an overall GPA and an upper-division GPA of 3.0 or better (B or better). Note: a B- average is equivalent to a GPA of 2.67, and therefore, it does not satisfy this graduation requirement. For more information on grades and grading policies, please visit:
`http://gradschool.ufl.edu/catalog/current-catalog/catalog-general-regulations.html#grades`

16. Make-up Exam Policy:

In the case a student is unable to attend the midterm or the final exam for a family or medical emergency, a make-up exam will be organized as soon as feasible for both the instructor and the student, provided that the instructor was given advanced notice of the situation. Students who miss an exam without advanced notice to the instructor (or without a valid reason for which such notice could not be given) will receive a F for the exam and will not be given a make-up exam.

17. Honesty Policy:

All students admitted to the University of Florida have signed a statement of academic honesty committing themselves to be honest in all academic work and understanding that failure to comply with this commitment will result in disciplinary action. This statement is a reminder to uphold your obligation as a UF student and to be honest in all work submitted and exams taken in this course and all others.

In particular, the material submitted by students in this class should reflect their work. If external resources such as books, individuals, websites, etc (other than textbook and/or the instructor) are consulted to answer homework or project problems, these external resources should be clearly acknowledged on the assignment. The use of external resources should be kept to a minimum. Further, it is expected that students will do their best to do their work on their own. If students feel "stuck" on a particular problem, it is considered acceptable to discuss it with other students of the class provided that this discussion is limited to general ideas and strategies about the problem (which can be thought of as "hints"). In particular, no written material should be exchanged. Assignments and test not satisfying the above requirements will be given a grade of F.

18. Accommodation for Students with Disabilities:

Students Requesting classroom accommodation must first register with the Dean of Students Office. That office will provide the student with documentation that he/she must provide to the course instructor when

requesting accommodation.

19. UF Counseling Services:

Resources are available on-campus for students having personal problems or lacking clear career and academic goals. The resources include:

- UF Counseling & Wellness Center, 3190 Radio Rd, 392-1575, psychological and psychiatric services.
- Career Resource Center, Reitz Union, 392-1601, career and job search services.

20. Software Use:

All faculty, staff and student of the University are required and expected to obey the laws and legal agreements governing software use. Failure to do so can lead to monetary damages and/or criminal penalties for the individual violator. Because such violations are also against University policies and rules, disciplinary action will be taken as appropriate. We, the members of the University of Florida community, pledge to uphold ourselves and our peers to the highest standards of honesty and integrity.