After-Tax Portfolio Optimization and The Observer Effect

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Classic before-tax portfolio optimization model of Grinold and Kahn (1999) is redone completely after-tax.

Portfolio of Taxpayer is a simple bond "Bond" and set of stocks holdings in some universe of stocks: \$s of face value x_0; and number of shares x_j for $j = 1, 2, \ldots, J$.

Combination of math, finance, accounting, & tax law.

Key impediment: can trade before-tax cash flows but not after-tax cash flows.

After-tax value of a portfolio at an instant in time: pile of cash then with all assets & liabilities of all kinds cashed out after-tax.

Given exogenous: portfolio $\overline{P}$; Tax Situation; $\overline{\text{Bond}}$ & universe of stocks; expected proportional change α_j in exogenous $\overline{p}_j$ from i_A to i_L, i.e., $p_j = \overline{p}_j\,(1+\alpha_j)$.

From perspective of now "i_A", the objective is "Adjustment" of portfolio to max expected value at period end i_L of pile, minus: a risk factor λ times variance of pile, relative to a benchmark portfolio.

Exogenous portfolio $\overline{P}$ has $\overline{\text{Bond}}$ and number of shares $\overline{x}_j$.

<u>Regular Payments occur at measurement dates</u>

At i_A: exogenous $\overline{\text{Bond}}$ payment $\overline{x}_0\,(1+\overline{s})$; endogenous Adjustment has shares sold $\max[0, \overline{x}_j - x_j]$ & shares purchased $\max[0, x_j - \overline{x}_j]$.

At i_L: endogenous Bond payment $x_0\,(1+s)$; & Liquidation of P has proceeds $p_j\,x_j$.

Loose Payments are those that can occur at other dates.

Estimated tax payments $\overline{\text{ETP}}$ stemming from Adjustment taxable events:

(a.) $\overline{\text{Bond}}$ interest income tax $T\ \overline{s}\ \overline{x}_0$, &

(b.) capital gain tax from sale of stocks in $\overline{P}$ is $G\left(\overline{p}_j - pp_j\right) \max[0, \overline{x}_j]$.

Estimated tax payments ETPs stemming from Liquidation taxable events

(a.) Bond interest income tax $T\, s\, x_0$, and

(b.) capital gain tax from liquidation of all P stocks
$G\left(p_j - pp_j\right) \max[0, x_j - \overline{x}_j]$.

Special Debt Instruments "sdis"

sdis transform Loose Payments from whenever they occur into a sdi fee paid on i_A or i_L. This fee cashes out all assets & obligations of the P and its Liquidation:

(a.) SDIbit transforms Bond's ordinary-income ETPs at their ETPDs into a fee Feebit paid at i_L.

(b.) SDIcgt_j transforms stock j capital-gain ETPs at their ETPDs into a fee Feecgt_j paid at i_L.

An sdi is a forward contract entered at i_A or i_L, in which counterparty offsets the Loose Payments in exchange for a sdi fee from or to Taxpayer at i_A or i_L, respectively.

But sdi creates distinct implicit interest-income taxable event, which generates ETPs, which sdi also offsets. We call Loose Payment and these Endogenous ETPS the "Assigned Payments"

Expected Valuation Random Variable

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Expected Valuation V at i_L is sum of Regular Payment, Loose Payment, and SDIs:

$$V \;=\; x_0\,(1+s) \;+\; \sum_{j=1}^{j=J} p_j\,x_j \;+\; \text{Feebit} \;+\; \sum_{j=1}^{j=J} \text{Feecgt}_j.$$

Model's objective Obj is V minus: risk-aversion factor λ times variance of V, which is σ_a^2.

Start with covariance matrix of the stocks j prices in universe and construct σ_a^2, which is the after-tax variance of V relative to such for $\tilde{P}$.

$$Obj \;=\; V \;-\; \lambda\,\sigma_a^2.$$

Observer Effect

To measure the after-tax value of P, we liquidate on i_L, whihc triggers complex taxable events that generate ETPs.

ETPS occur on subsequent ETPDS. SDIs transform each set of ETPs into a pile at i_L.

But SDIs are foreign bodies that create their own distinct taxable events, generating distinct ETPs, which change the after-tax value.

Observer Effect is Tax on SDI's implicit interest income (Ibit and Icgt_j), which are components of the sdi fees:

$$T \left[\text{Feebit} + \text{Ibit} + \sum_{j=1}^{j=J} (\text{Feecgt}_j + T\,\text{Icgt}_j) \right], \text{ where}$$

Ibit and Icgt_j are the Taxpayer's income from SDIbit and SDIcgt_j themselves. This income is found by viewing all the cash flow of the sdi as a loan. It is just the undiscounted sum of the fee and the "Assigned Payments" of the SDI.