CASE STUDY: Traveling Salesman Problems (linear (TSP (matrix_of_distances))

Background

This case study solves several Travelling Salesman Problems (TSPs). PSG has TSP operator for a compact formulation of problems. The length of the path which goes through all vertices of the graph is calculated with the operator "linear(TSP(matrix_of_distances))". For instance, if you want to find the shortest path you can write the following PSG problem statement:

```
minimize
  linear(TSP(matrix_of_distances))
```

TSP optimization problems can be solved only if Gurobi package is installed at the computer. PSG is using Gurobi-based solvers CarGrb or VanGrb for solving problems with TSP operator.
Problems were solved with data downloaded from the link [1].

Matrix of distances between nodes

A matrix of distances between nodes has names of nodes in the first (header) row. Every column contains distances between the node identified in the header field and all other nodes. An order of nodes in every column is defined by the order of nodes in the first row of the matrix. PSG uses data above diagonal of the matrix and ignores data on and below the diagonal. Matrix may have the full or sparse format. If matrix has the sparse format, then absent entries are considered as absent edges (or edges with `+infinity` distance). Values in the matrix may be positive, zero, or negative. Matrix should contain at least 3 node names.

References

[1] http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/

Further we provide mathematical problem statement for the Plain TSP.

Find a tour through a graph with a minimal path lenth

$$\min \sum_{e \in V} c_e x_e \qquad (1)$$

subject to

Constraint on connection of every node

$$\sum_{e \in E_i} x_e = 2, \;\; i \in I, \qquad (2)$$

Constraint on connectivity of all nodes

$$\max_S \left\{ 2 - \sum_{e \in C(S)} x_e \mid S \subset I, 1 < |S| < n \right\} \le 0, \qquad (3)$$

Boolean condition on variables

$$x_e \in \{0,1\}, \qquad e \in E. \tag{4}$$

Notations

$G = (I, V)$ = graph;
I = set of nodes;
n = number of nodes;
V = set of edges;
c_e = weight of edge $e \in E$;
V_i = set of edges incident to node $i \in I$;
S = subset of nodes, $S \subset I$;
$C(S)$ = cut-set of graph dividing nodes on S and $I \backslash S$ subsets, $C(S) \subset V$.

Optimization Problem 1

Solve plain TSP

$$\min \ linear(TSP(matrix\_of\_distances)).$$

Optimization Problem 2

Find tour in a graph with a maximal weight

$$\max \ linear(TSP(matrix\_of\_distances)).$$

Optimization Problem 3

Solve TSP with a constraint on budget

$$\min \ linear(TSP(matrix\_of\_distances))$$

subject to

Constraint on budget

$$linear(matrix\_TSP\_budget) \leq const1.$$

Optimization Problem 4

Minimize budget allocated for "building roads" with a constraint on the length of full tour

$$\min \ linear(matrix\_TSP\_budget)$$

subject to

Constraint on the length of tour

$$linear\big(TSP(matrix\_of\_distances)\big) \leq const2.$$