

Systemic Risk and Renewable Energy

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- We explore the relationship between systemic risk in an electricity market with significant renewable generation capacity in a highly simplified model

Illustration: A Hydro-based Market

- A purely hydro-based market with $I > 0$ units of energy in storage and two stages.
- If x units are released in the first stage then

$$q = \max\{I - x + w, 0\}$$

units are available for consumption in the second stage where $w \geq 0$ is a random water inflow (with distribution F in $[0, \bar{w}]$).

Illustration: A Hydro-based Market

- Assume inelastic demand $d > 0$ in each stage and aggregate thermal capacity $k_T < d$ at marginal cost c_T .
- We assume $I + \bar{w} < d$, that is, thermal generation must be used in the second stage w.p. 1.
- Assuming an outage cost $v > c_T$ the total cost (over two stages) of releasing x in the first stage is

$$C(x) = c_T(d - x) + c_T \max\{d - q, k_T\} + R(x)$$

where

$$R(x) = v \max\{d - k_T - q, 0\}$$

is the outage cost.

Illustration: A Hydro-based Market

- Assuming $q > 0$, rationing takes place whenever

$$w < w(x) \triangleq d - k_T - I + x$$

- The expected value of total cost (over two stages) is:

$$\begin{aligned} E[C(x)] &= c_T(d - x) + c_T \int_{w(x)} (d - I + x - w) dF(w) \\ &\quad + v \int_0^{w(x)} (d - k_T - I + x - w) dF(w) \end{aligned}$$

- The probability the rationing cost exceeds a threshold r is

$$\begin{aligned}\Pr(R(x) > r) &= \Pr(v(d - k_T - q) > r) \\ &= \Pr(q < d - k_T - \frac{r}{v}) \\ &= \Pr(w < d - k_T - \frac{r}{v} + x - l) \\ &= F(d - k_T - \frac{r}{v} + x - l)\end{aligned}$$

- $VaR_\alpha(x)$ is defined as

$$F(d - k_T - \frac{VaR_\alpha(x)}{v} + x - l) = 1 - \alpha$$

or equivalently,

$$VaR_\alpha(x) = v(d - k_T + x - l - F^{-1}(1 - \alpha))$$

- The conditional value at risk

$$\begin{aligned} CVaR_{\alpha}(x) &= E[R(x) | R(x) > VaR_{\alpha}(x)] \\ &= \frac{1}{\alpha} \int_0^{\alpha} VaR_y(x) dy \\ &= v(d - k_T + x - l - \frac{1}{\alpha} \int_0^{\alpha} F^{-1}(1 - y) dy) \end{aligned}$$

- A risk averse benevolent planner would solve

$$\begin{aligned} \min_{x \in [0, l]} \quad & E[C(x)] \\ \text{s.t} \quad & \\ & CVaR_{\alpha}(x) \leq \bar{R} \end{aligned} \tag{1}$$

- Let $\mu^* \geq 0$ denote the Lagrange multiplier, (1) can be written as

$$\min_{x \in [0, I]} [E[C(x)] + \mu^* (CVaR_\alpha(x) - \bar{R})]$$

- First order condition is:

$$c_T = (v - c_T)F(d - k_T - I + x^*) + \mu^* v \quad (2)$$

- Assume the water in store is distributed accross N reservoirs each with level l_i ($\sum_i l_i = I$).
- In a market dispatch each reservoir determines independently how much to release $x_i \geq 0$ and individual inflow is $w_i \geq 0$.
- We assume $w_i = \rho_i w$ donde $\rho_i \in (0, 1)$ y $\sum_i \rho_i = 1$. The individual balance equation is $q_i = \max\{l_i - x_i + w_i, 0\}$.

- Let $q = \sum_i q_i$. The spot price in the second stage is:

$$\tilde{p}_2 = \begin{cases} 0 & q \geq d \\ c_T & q \in (d, d - k_T) \\ v & q < d - k_T \end{cases}$$

- The profits for firm i are given by

$$\Pi_i = p_1 x_i + \tilde{p}_2 q_i$$

- The probability of outage is associated with the even

$$\sum_{i=1}^N q_i < d - k_T$$

when $q_i > 0$ this is equivalent to

$$\sum_{i=1}^N (l_i - x_i + w_i) = \sum_{i=1}^N (l_i - x_i) + w < d - k_T$$

- Thus outage occurs whenever $w < w(x)$.

- Expected profit is

$$\begin{aligned} E[\Pi_i] &= p_1 x_i + E[\tilde{p}_2 q_i] \\ &= p_1 x_i + c_T \int_{w(x_i, x_{-i})} (l_i - x_i + \rho_i w) dF(w) \\ &\quad + v \int_0^{w(x_i, x_{-i})} (l_i - x_i + \rho_i w) dF(w) \end{aligned}$$

- Suppose each firm i aims to maximize expected profit.
- The first order condition is

$$p_1 = (v - c_T) F(d - k_T - \sum_{i \in I} (l_i - x_i^*)) \quad (3)$$

- Comparing (2) and (3) we see that if $\mu^* = 0$ the market dispatch equals the optimal dispatch if $p_1 = c_T$
- If $\mu^* > 0$ the market delivers the optimal risk-constrained dispatch iff

$$p_1 = c_T - \mu^* v$$

- If $p_1 > c_T - \mu^* v$ then the market dispatch increases systemic risk (the firms release too much water in the first period)

- Suppose an “uplift” of $\mu^* v$ is charged per unit in the first stage and in the second stage prices are $\tilde{p}'_2 = \tilde{p}_2 + \mu^* v$
- The first order condition is

$$p_1 - \mu^* v = (v - c_T) F(d - k_T - \sum_{i \in I} (l_i - x_i^*))$$

- In a highly simplified model, we have shown how increased renewable capacity is likely to induce additional systemic risk in the operation of electricity markets.
- A simple “uplift” can provide the right incentives.
- However, in most electricity markets intermittency is heterogeneous. This implies differential effects on systemic risk.
- A topic for future research.