Simulating future test and redesign considering epistemic model uncertainty

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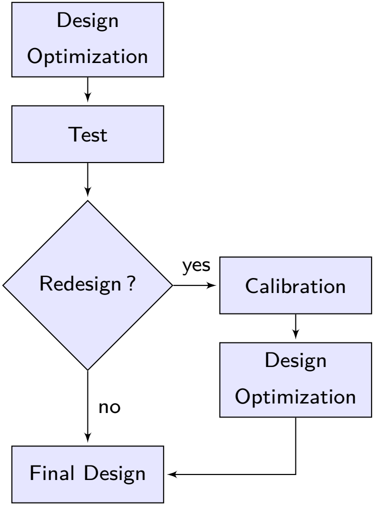
Presentation outline

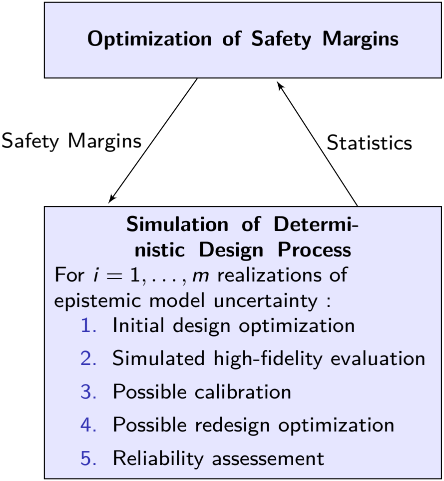
Design, test, and possible redesign

- At the initial design stage, design optimization often relies on low-fidelity models
- In the future, high-fidelity models can be used to test the initial design
- High-fidelity model results may trigger a redesign process
- Redesign can restore safety or improve design performance
  - May delay design process and/or increase costs



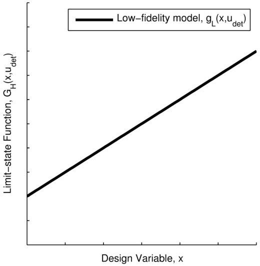
Optimization framework

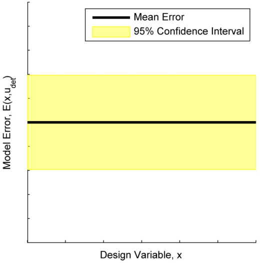
- The design process is formulated deterministically in terms of safety margins
- A simulation of model error explores how the optimum deterministic design, reliability, and performance may change conditional on the future test
- The safety margins are optimized based on the design process simulation

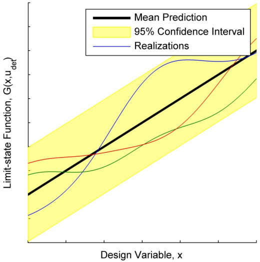


Multi-fidelity modeling

Low-fidelity + **Error** = **Predictive Model**







Modeling epistemic model error

The high-fidelity model $g_H(\cdot,\cdot)$ is predicted from the low-fidelity model $g_L(\cdot,\cdot)$ as

$$\hat{G}_H(\boldsymbol{x},\boldsymbol{u}) = g_L(\boldsymbol{x},\boldsymbol{u}) + \hat{E}(\boldsymbol{x},\boldsymbol{u})$$

where function $\hat{E}(\boldsymbol{x},\boldsymbol{u})$ is a Gaussian process (GP) model

Optimization of safety margins

- The safety margins are optimized to minimize expected cost while satisfying constraints on expected probability of failure and probability of redesign
- The constraint on probability of redesign can be varied to capture the tradeoff between performance and probability of redesign

Safety margin optimization

$$
\begin{aligned}
\min_{\boldsymbol{n}} \quad & E\left[f(\hat{\boldsymbol{X}}_{final})\right] \\
\text{s.t.} \quad & E\left[\hat{P}_{f,final}\right] \leq \bar{p}_f \\
& p_{re} \leq \bar{p}_{re}
\end{aligned}
$$

- $E[\cdot]$: expected value with respect to epistemic uncertainty
- $\hat{\boldsymbol{X}}_{final}$: possible final designs
- $\hat{P}_{f,final}$: possible final probability of failures
- p_{re} : probability of redesign
- $\boldsymbol{n} = \{n_{ini}, n_{lb}, n_{ub}, n_{re}\}$: vector of safety margins

Initial design optimization

- During design optimization aleatory variables are replaced with conservative values and mean surrogate prediction is used
- The safety margin, n_{ini}, changes the size of the feasible design space to add more or less conservativeness to the design
- A smaller safety margin may allow for better design performance but final design may be less safe

Initial design optimization

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & E\left[\hat{G}_H(\boldsymbol{x}, \boldsymbol{u}_{det})\right] - n_{ini} > 0 \end{aligned}$$

- $E\left[\hat{G}_H(\boldsymbol{x}, \boldsymbol{u}_{det})\right]$: mean surrogate prediction
- $\boldsymbol{u}_{det}$: vector of fixed conservative values used in place of aleatory random variables
- n_{ini} : initial safety margin

Future test and redesign

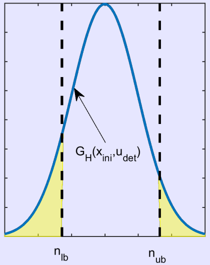
- The design will be tested under fixed conditions $\boldsymbol{U} = \boldsymbol{u}_{det}$
- The uncertainty in the test result is only due to the epistemic model uncertainty (not aleatory uncertainty $\boldsymbol{U}$)
- A realization of a possible test result (i.e. high-fidelity evaluation) is simulated based on the GP error model
- If the test is passed the designer will accept the initial design as being satisfactory
- If the test is not passed, redesign will be performed to find a new design

Redesign decision

The test will be passed if

$$n_{lb} \leq \hat{g}_H^{(i)}(\boldsymbol{x}_{ini}, \boldsymbol{u}_{det}) \leq n_{ub}$$

where $\hat{G}_H(\boldsymbol{x}_{ini}, \boldsymbol{u}_{det})$ is normally distributed according to GP model



Calibration and redesign optimization

- If redesign is required, the model is first calibrated conditional on the realization of the test result
- The model is calibrated by adding the simulated data point to the initial design of experiment (DoE)
- A new design $\hat{\boldsymbol{x}}_{re}^{(i)}$ is found by solving a new optimization problem
- During redesign the feasible design space is changed in two ways
  1. Redesign safety margin n_{re} may be different than n_{ini}
  2. Mean surrogate prediction is updated conditional on the realization of future test

Redesign optimization

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & E\left[\hat{G}_{H,upd}^{(i)}(\boldsymbol{x}, \boldsymbol{u}_{det})\right] - n_{re} > 0 \end{aligned}$$

- $E\left[\hat{G}_{H,upd}^{(i)}(\boldsymbol{x}, \boldsymbol{u}_{det})\right]$: updated mean surrogate prediction
- n_{re} : redesign safety margin

Probability of failure calculation

- After possible redesign, the probability of failure is calculated with respect to aleatory uncertainty
- The probability of failure is calculated for each possible realization of the GP model
- The probability of failure after redesign is different from the initial probability of failure because we will have the opportunity to change the design after the test

Probability of failure calculation

Initial probability of failure

$$\hat{p}_{f,ini}^{(i)} = Pr_U\left[\hat{g}_H^{(i)}(\boldsymbol{x}_{ini}, \boldsymbol{U}) < 0\right]$$

Probability of failure after redesign

$$\hat{p}_{f,re}^{(i)} = Pr_U\left[\hat{g}_H^{(i)}(\boldsymbol{x}_{re}^{(i)}, \boldsymbol{U}) < 0\right]$$

- $Pr_U[\cdot]$: probability with respect to aleatory uncertainty

Final distributions

- Finally, the distributions of the design variable and probability of failure are obtained
- The redesign decision shapes the distribution of possible future designs and probability of failure
- The bounds on acceptable safety margins n_{lb} and n_{ub} ensure that an unsafe or overly conservative design does not make it through the testing process

Final distributions

Let $q^{(i)}$ denote an indicator function for the redesign decision

$$q^{(i)} = \left\{ \begin{array}{ll} 0 & \text{No redesign} \\ 1 & \text{Redesign} \end{array} \right\}$$

Final design after possible redesign

$$\hat{\boldsymbol{X}}_{final} = (1 - Q)\boldsymbol{x}_{ini} + Q\hat{\boldsymbol{X}}_{re}$$

Final probability of failure after possible redesign

$$\hat{P}_{f,final} = (1 - Q)\hat{P}_{f,ini} + Q\hat{P}_{f,re}$$

Demonstration example

- A cantilever beam is designed to minimize mass subject to a constraint on tip displacement
- There are 4 variables : 2 design variables, 2 aleatory random variables

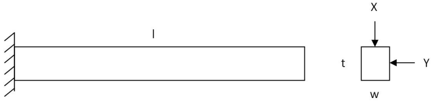
Problem parameters :

Design variables : $\boldsymbol{x} = \{w, t\}$

- $3 \leq w \leq 8$ in : width of beam
- $3 \leq t \leq 8$ in : thickness of beam

Aleatory variables : $\boldsymbol{U} = \{X, Y\}$

- $X \sim \text{Normal}(6250, 1250^2)$ lbs : vertical tip load
- $Y \sim \text{Normal}(12500, 1250^2)$ lbs : horizontal tip load
- $\boldsymbol{u}_{det} = E(\boldsymbol{U}) + 1.645\sqrt{Var(\boldsymbol{U})} = \{8306, 14556\}$ lbs



Limit-state function

- The low-fidelity model is based on Euler-Bernoulli beam theory and doesn't account for shear stress effects that occur in short, stubby beams
- The high-fidelity model is based Timoshenko beam theory
  - Model is only used for generating preliminary test data
- The error model is constructed based on preliminary test data
  - 4 beam designs (corners)
  - 9 loading configurations (3 levels)

Problem parameters :

Low-fidelity model

$$g_L(\boldsymbol{x}, \boldsymbol{U}) = \bar{d} - \frac{4l^3}{ewt}\sqrt{\left(\frac{Y}{t^2}\right)^2 + \left(\frac{X}{w^2}\right)^2}$$

High-fidelity model

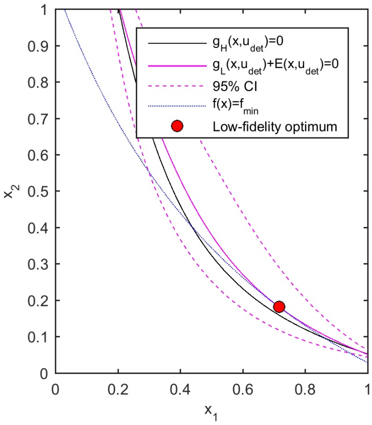
$$g_H(\boldsymbol{x}, \boldsymbol{U}) = \bar{d} - \sqrt{d_x^2 + d_y^2}$$

$$d_x = \left(\frac{3lX}{2gwt} + \frac{4l^3X}{ewt^3}\right) \quad d_y = \left(\frac{3lY}{2gwt} + \frac{4l^3Y}{ew^3t}\right)$$

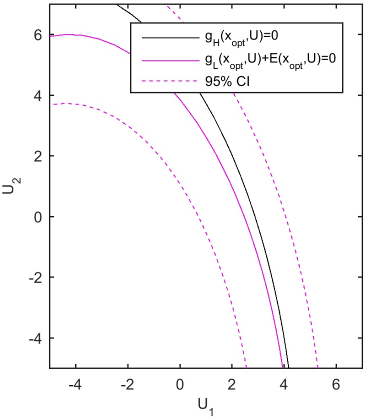
- $l = 8$: length of beam (in)
- $e = 29 \times 10^6$: elastic modulus (psi)
- $g = 11.2 \times 10^6$: shear modeulus (psi)
- $\bar{d} = 2 \times 10^{-3}$ allowable tip displacement (in)

Visualizing model uncertainty

Deterministic design optimization
(normalized design space)

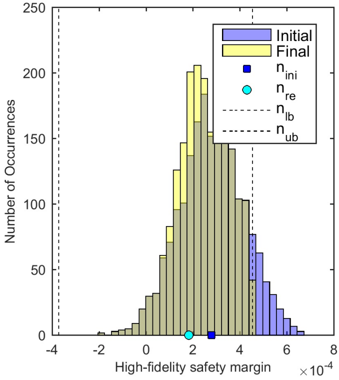


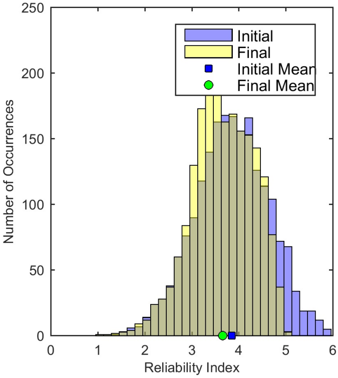
Reliability analysis
(standard normal space)



Safety margin and reliability distribution

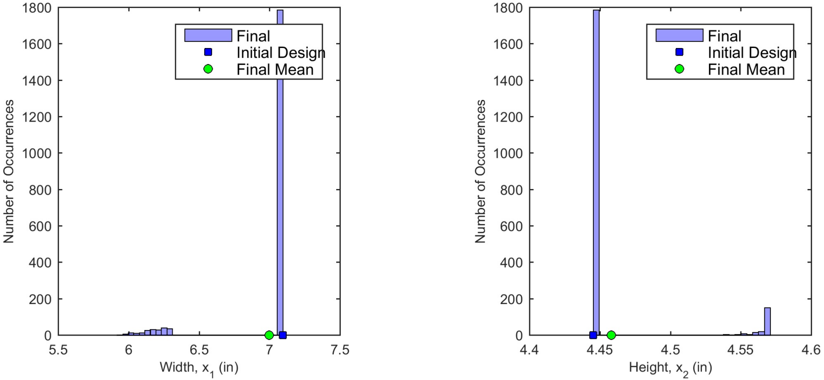
- Example of using redesign to improve performance if initial design is revealed to be overly conservative





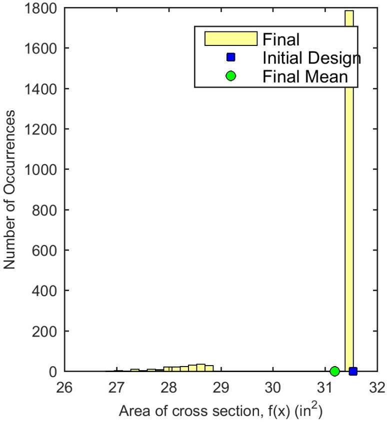
Design variable distribution

- If redesign is performed, a thinner width of beam is selected to reduced mass



Area of cross section distribution

- If redesign is required, the area of the beam is reduced by about 10%
- This is a relatively large reduction for weight critical applications
- In this example, a designer may be willing to accept the risk of redesign since the potential performance improvement is large



Discussion & Future Work

- Preliminary results illustrate the benefits of simulating a future test and redesign to make informed decisions at the initial design stage
- In this study, we introduced the Gaussian Process error model as a flexible representation of epistemic model error
  - Previous, work by UF MDO group had relied on the assumption of constant model bias
- In future work we would like to calculate the tradeoff curve for expected performance and probability of redesign
- Future work will also compare the simulation results to the results that are obtained if we perform a true test using the high-fidelity model
- Final results will be presented at the AIAA SciTech Forum 2016 in San Diego on January 4-8th 2016