CASE STUDY: Linear Regression in Tranche (ltranche, meansquare_err, meanabs_pen)

Background

An insurance company buys some protection from a reinsurance company. This case study solved a linear regression problem for building an optimal reinsurance contract (from the point of view of the insurance company buying reinsurance). The contract specifies loading coefficients for losses and attachment and detachment points of a tranche. The insurance company obtains reimbursement if the linear combination of the losses “hits” the tranche. The reimbursement equals the difference between the linear combination and the attachment point, under condition that the linear combination exceeds the attachment point. However the reimbursement does not exceed the width of the tranche (which is the difference between detachment and attachment points). The insurance company wants to match its losses projected to the tranche and reimbursements from the contract. In summary, the insurance company wants to find an optimal linear combination of losses in the contract specification to get a good protection in the specified tranche.

Notations

$\vec{x} = (x_1, \dots, x_I)$ = vector of decision variables;

$G_j^0(\vec{x}) = \sum_{i=1}^{I} \theta_{ji} \cdot x_i$ = gain (or predicted value by linear regression) on scenario, $j = 1, \dots, J$;

b_j^0 = scenario benchmark on scenario, $j = 1, \dots, J$;

l =attachment point of a tranche;

u =detachment point of a tranche;

$G_j(\vec{x}) = \min\left(u - l, \max\left(0, G_j^0(\vec{x}) - l\right)\right)$ = gain (or predicted value by linear regression) projected to tranche on scenario, $j = 1, \dots, J$;

$b_j = \min\left(u - l, \max\left(0, b_j^0 - l\right)\right)$ = scenario benchmark projected to tranche on scenario, $j = 1, \dots, J$;

p_j = probability of scenario, $j = 1, \dots, J$;

$L_j(\vec{x}) = b_j - G_j(\vec{x})$ = Loss (or residual of regression) for projected on tranche scenario, $j = 1, \dots, J$; this Loss is coded in PSG by $ltranche$ function with two input matrices: 1) matrix with values (θ_{ji}) and b_j^0 ; 2) matrix with l and u ;

$L(\vec{x})$ = random Loss function with scenarios $L_j(\vec{x})$, $j = 1, \dots, J$;

$meansquare\_err(L(\vec{x})) = \sum_{j=1}^{J} p_j L_j^2(\vec{x})$ = PSG function Mean Squared Error;

$meanabs\_pen(L(\vec{x})) = \sum_{j=1}^{J} p_j \left|L_j(\vec{x})\right|$ = PSG function L1 Error (called Mean Absolute Penalty).

Optimization Problem 1

Minimize Mean Squared Error

$$minimize\ \ meansquare\_err\left(L(\vec{x})\right)$$

Optimization Problem 2

Minimize L1 error (Mean Absolute Penalty)

$$minimize\ \ meanabs\_pen\left(L(\vec{x})\right)$$