CASE STUDY: Data Envelopment Analysis

Background

This case study compares the relative managerial efficiency of five companies by applying the CCR Model from Data Envelopment Analysis (DEA) (Charnes et al, 1978).

The comparison is made for the companies, Sempra Energy, American Electric Power Co., Inc., AGL Resources Inc., CenterPoint Energy, Inc. and Duke Energy Corporation from Utilities industrial sector. We used Earnings per Share, Debt to Equity Ratio, Leverage Ratio, Solvency Ratio as inputs and Net Profit Margin and Price to Earnings Ratio as outputs (collected from the financial statements of these companies). The categorization of inputs and outputs was made according to Zhang, 2007.

The model maximize the ratio of the weighted outputs and the weighted inputs of the company for every company, subject to constraints prohibiting that the ratio of the other companies to be higher than 1.

For further readings about DEA, consult "*Handbook on data envelopment analysis"* from Cooper and "*Service Productivity Management: Improving Service Performance Using Data Envelopment Analysis (DEA)"* from Sherman.

The optimization problem is solved with two equivalent formulations utilizing the Multilinear (problem 1) and Max_Risk PSG (problem 2) functions. Each of these problems should be run five times to estimate each company. The case study demonstrates how to solve these problems for five companies in one shot using PSG cycles. Problem 3 solves the optimization problem for five companies in one shot utilizing the cycle and the Multilinear PSG function. Problem 4 solves the optimization problem for five companies in one shot utilizing the cycle and the Max_Risk PSG function.

References

Charnes, A., Cooper, W. W. , and E. Rhodes (1978): "Measuring the efficiency of decision making units." *European Journal of Operational Research*.

Cooper, W.W, Lawrence M. S, and Z. Joe. (2011): "*Handbook on data envelopment analysis"*. Springer.

Sherman, H. D. (2006): "*Service Productivity Management: Improving Service Performance Using Data Envelopment Analysis (DEA). Includes DEAFrontier Software"*. Springer.

Zhang, X. (2007): "The Generalized DEA Model of Fundamental Analysis of Public Firms, with Application to Portfolio Selection.". Doctoral Dissertation, University of Tennesse - Knoxville

Notations

I = number of outputs; $i = \{1, \ldots, I\}$ index of outputs;

J = number of inputs; $j = \{1, \ldots, J\}$ index of inputs;

K = number of companies; $k = \{1, \ldots, K\}$ index of companies;

u_i = weight of the output i;

v_j = weight of the input j;

γ_{ik} = amount of the output i produced by the company k;

θ_{jk} = amount of the input j consumed by the company k;

$L_k(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta}) = -\sum_{i=1}^{I} u_i \gamma_{ik} + \sum_{j=1}^{J} v_j \theta_{jk}, \ k = 1, \ldots, K$;

$\max_risk(L_1(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta}), \ldots, L_k(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta})) = \max_{1 \le k \le K} L_k(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta})$;

Summary of notations: Each company k has input parameters numbered by index i and produces output parameters numbered by index j. Each output γ_{jk} and input θ_{ik} has a non-negative weight denoted by u_i and v_j respectively.

Optimization Problem 1 (Linear Programming Formulation)

maximizing relative efficiency of company k=0

$$max \sum_{i=1}^{I} u_i \gamma_{i0} \tag{1}$$

subject to

constraint ensuring that for all companies efficiency is not higher than 1

$$\sum_{i=1}^{I} u_i \gamma_{ik} - \sum_{j=1}^{J} v_j \theta_{jk} \le 0, \quad k = 0,1, \ldots, K \tag{2}$$

constraint maintaining denominator of ratio for company k = 0 equal to 1

$$\sum_{j=1}^{J} v_j \, \theta_{j0} = 1 \tag{3}$$

lower bounds on weights

$$u_i, v_j \geq 0 \,, \quad i = 1, \ldots, I, \; j = 1, \ldots, J \tag{4}$$

Optimization Problem 2 (Equaivalent Formulation with Max_Risk Function)

maximizing relative efficiency of company k=0

$$max \sum_{i=1}^{I} u_i \, \gamma_{i0} \tag{5}$$

subject to

constraint ensuring that for all companies efficiency is not higher than 1

$$max_risk\left(L_1(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta}), \ldots, L_k(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta})\right) \leq 0, \tag{6}$$

constraint maintaining denominator of ratio for company k = 0 equal to 1

$$\sum_{j=1}^{J} v_j \, \theta_{j0} = 1 \tag{7}$$

lower bounds on weights

$$u_i, v_j \geq 0 \,, \quad i = 1, \ldots, I, \; j = 1, \ldots, J \tag{8}$$

Optimization Problem 3 (Linear Programming Formulation in one shot)

For 5 problens

maximizing relative efficiency of company k=0

$$max \sum_{i=1}^{I} u_i \, \gamma_{i0} \tag{9}$$

subject to

constraint ensuring that for all companies efficiency is not higher than 1

$$\sum_{i=1}^{I} u_i \gamma_{ik} - \sum_{j=1}^{J} v_j \theta_{jk} \leq 0, \quad k = 0,1, \ldots, K \tag{10}$$

constraint maintaining denominator of ratio for company k = 0 equal to 1

$$\sum_{j=1}^{J} v_j \theta_{j0} = 1 \tag{11}$$

lower bounds on weights

$$u_i, v_j \geq 0 , \quad i = 1, \ldots, I, \ j = 1, \ldots, J \tag{12}$$

End For

Optimization Problem 4 (Equaivalent Formulation with Max_Risk Function in one shot)

For 5 problens

maximizing relative efficiency of company k=0

$$max \sum_{i=1}^{I} u_i \gamma_{i0} \tag{13}$$

subject to

constraint ensuring that for all companies efficiency is not higher than 1

$$max_risk\left(L_1(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta}), \ldots, L_k(\vec{u}, \vec{v}, \vec{\gamma}, \vec{\theta})\right) \leq 0, \tag{14}$$

constraint maintaining denominator of ratio for company k = 0 equal to 1

$$\sum_{j=1}^{J} v_j \theta_{j0} = 1 \tag{15}$$

lower bounds on weights

$$u_i, v_j \geq 0 , \quad i = 1, \ldots, I, \ j = 1, \ldots, J \tag{16}$$

End For