CASE STUDY: Calibrating Risk Preferences (meanabs_pen, linear)

Background

This case study extracts risk preferences of investors by solving a linear regression model with linear constraints on coefficients. "Risk preferences" are expressed by a risk functional (a deviation measure), which is used by an investor for measuring risk and solving portfolio optimization problems. Contrary to the classical Markowitz portfolio theory, where investors measure risk by standard deviation, this case study assumes that the unknown deviation measure belongs to a class of Mixed CVaR Deviations. In particular, we consider the case when the Mixed CVaR Deviation is a weighted average of the following five CVaR Deviation terms with confidence levels 50%, 75%, 85%, 95%, and 99% (theory and description of this case are available in Kalinchenko et al (2012)). The Mixed CVaR Deviation has five weighting parameters (lambdas), which are nonnegative and sum up to 1. These lambda coefficients are estimated by matching the market option prices with prices expressed via generalized CAPM pricing relations. Matching is done by minimizing a L1 (the error term is sum of absolute values of the differences between market and calculated prices).

References

- Kalinchenko, K., Uryasev, S. and R.T. Rockafellar (2012). Calibrating risk preferences with generalized CAPM based on mixed CVaR deviation. *Journal of Risk*, 15(1), pp. 1–26.

Notations

S = total number of scenarios;
s = index of scenarios $\{1,\ldots, S\}$;
J = number of options;
j = option index, $1 \le j \le J$;
K_j = option j strike price;
P_K = market price of an option with strike K;
$r_I^{(s)}$ = index return in scenario S;
r_0 = risk-free rate of return;
I_0 = index price;
$\zeta_K^{(s)}$= payoff of option with strike K in scenario s;
$r_K^{(s)}$= return of option with strike K in scenario s;
L = number of terms in the CVaR deviation (L= 5);
l = term index, $1 \le l \le L$;
α_l = percentiles, defining tails of distribution (99%. 95%, 85%, 75% and 50%);
λ_l = coefficient for term l in CVaR deviation;
$E[\cdot]$ = mathematical expectation, $E[x] = \frac{1}{S}\sum_{s=1}^{S} x^{(s)}$;
$E[\cdot\,|condition]$ = conditional mathematical expectation, $E[x] = \frac{\sum_{s=1}^{S} x^{(s)}\mathrm{Ind}\{condition \text{ holds for } s\}}{\sum_{s=1}^{S}\mathrm{Ind}\{condition \text{ holds for } s\}}$,

where Ind{true} = 1 and Ind{false} = 0;

$VaR_\alpha[\cdot]$ = Value-at-Risk; $VaR_\alpha(x) = -y^{(\lfloor(1-\alpha)S\rfloor)}$, where y denotes elements of x ordered in ascending order;
$CVaR_\alpha^\Delta[\cdot]$ = CVaR (Conditional Value-at-Risk) deviation; $CVaR_\alpha^\Delta(x) = E[Ex - x|x < -VaR_\alpha(x)]$;
$A = \{A_j^l\}$ = design matrix, $A_j^l = \left(Er_{K_j} - r_0\right) CVaR_{\alpha_l}^\Delta(r_I) - (Er_I - r_0)E[Er_{K_j} - r_{K_j}|r_I < -VaR_{\alpha_l}(r_I)]$;

$\mathrm{linear}\left(\vec{\lambda}\right) = \sum_{l=1}^{L}\lambda_l$ = sum of lambdas;

meanabs_pen$(\vec{z}) = \frac{1}{|\vec{z}|}\sum|z_i|$ = L_1 norm;

meanabs_pen$(A \cdot \vec{\lambda})$ = norm of the residual in regression with design matrix A and coefficients λ_l.

Optimization Problem

minimizing L1 residual

$$\min_{\lambda} \quad \text{meanabs\_pen}(A \cdot \vec{\lambda}) \tag{CS.1}$$

subject to

$$\text{linear}(\vec{\lambda}) = \sum_{l=1}^{L} \lambda_l = 1 \tag{CS.2}$$

$$0 \leq \lambda_l \leq 1, \quad \forall l = 1, \dots, L \tag{CS.3}$$

Data Preparation for Case Study

Options trading data were carefully processed before conducting the case study. We included in the case study only options which are frequently traded. The prices of options were imported from the Wharton Business School database in .csv (comma separated) format. We processed these data and saved in .mat (MATLAB data file) format. The .m file, performing this procedure is provided. Note: when you run MATLAB code the subfolder \Data should be created in the case study folder, and the file IndexReturns.mat must be in this subfolder.

CS_CalibratingRiskPreferences.m file contains the MATLAB code for calibrating risk preferences for multiple dates. For every date it produces an output, containing coefficients in the CVaR deviation, GCAPM prices in dollars, market prices and GCAPM prices in implied volatility format, and some additional parameters. All results are recorded in OUTPUTTABLE.

CS_CalibratingRiskPreferences_ExtractData.m file contains the MATLAB code for extracting options market prices from a .csv file. Extracted data are saved in .mat files (Data\OptionsData <number>.mat) separately for each date. The set of dates is saved in a separate file Data\DatesSet.mat .

OptionPrices.csv file contains raw options data, imported from the Wharton Business School database.

The file problem_find_lambda.txt contains the description of a linear regression optimization problem for calibrating risk preferences for one date. The relevant data are in file matrix_regmatrixscenarios.txt. The file matrix_sum.txt contains the input for the constraint (sum of lamdas equals to one). The problem statement and data files can be run with Run-File Text PSG program, or they can be imported to other PSG Environments (MATLAB, Shell, or C++).

The file IndexReturns.mat contains returns on S&P500 over 12 years.