CASE STUDY: Optimal Allocation of Stock Levels and Stochastic Customer Demands to a Capacitated Resource (avg_pm_pen_ni, linear)

Background

This case study optimizes an allocation of stochastic demands to a capacitated resource.

This case study is based on the paper by Chen, S. and J. Geunes (2010). A single-period model is defined to solve a joint customer demand allocation and multiple-item stock level problem for a resource that must respond to uncertain customer demands. It is assumed that customer demands are statistically independent and that the demands for items are revealed upon being visited by a repair person (after problem diagnosis). There are m potential customers; the supplier must choose a subset of customers to serve using a capacity constrained resource (e.g., a vehicle) and the optimal resource stock level for each item. It is assumed that during a given customer-service visit, any item required for the customer's service that is not available results in an item-specific penalty cost for an inability to complete service. For each item carried on the vehicle, a variable cost is incurred (e.g., for loading/unloading and/or transporting the part). Because the vehicle stocking decisions must be determined prior to actual customer demand realizations, it may be practical in some contexts to consider a salvage value for each unused item carried on the vehicle (this might correspond to a reduction in future loading/unloading costs; alternatively, a negative value would correspond to an opportunity cost of the vehicle capacity usage). A customer-specific revenue is gained for each customer visit. The objective is to maximize the expected profit, or equivalently, to minimize the expected cost (equal to the negative of expected profit). We state our objective in minimization form and refer to this objective as the expected cost.

References

- Chen, S. and J. Geunes (2010): Optimal Allocation of Stock Levels and Stochastic Customer Demands to a Capacitated Resource. Submitted for publication.

Notations

i = item index, $i = 1, ..., m$;
j = customer index, $j = 1, ..., n$;
e_i = net penalty cost incurred for not satisfying a unit of demand for item i;
$S = \sum_{i=1}^{m} e_i$;
c_i = net variable cost for carrying item i in the vehicle;
π_j = net revenue gained by allocating customer j to the vehicle;
s_i = unit size of item i;
V = vehicle capacity;
d_{ij} = random normal distributed customer demand representing the number of units of item i that will be needed by customer j, described by mean μ_{ij} and standard deviation σ_{ij};
x_j = binary decision variable, equal to 1 if customer j is assigned to the vehicle, 0 otherwise;
$x = (x_1, x_2, \cdots, x_n)$ = vector characterizes the assignment of customers to the vehicle;
y_i = nonnegative decision variable equals to the number of units of item i carried in the vehicle;
$y = (y_1, y_2, \cdots, y_m)$ = vector determining the assignment of items to the vehicle;
$L_i(x, y_i) = \sum_{j=1}^{n} d_{ij}x_j - y_i$ = value of random loss function for item i = unsatisfied total demand on item i;
$avg\_pm\_pen\_ni_0(L_1(x, y_1), L_2(x, y_2), \ldots, L_m(x, y_m)) = \sum_{i=1}^{m} \frac{e_i}{S} * \mathrm{E}\left[\left(\sum_{j=1}^{n} d_{ij}x_j - y_i\right)^+\right]$ = Average Partial Moment Penalty Normal Independent for Loss with threshold 0 = weighted mean penalty incurred by not satisfying demands with weights e_i.

Optimization Problem 1

minimizing expected cost

$$\min_{x,y}\left\{S \cdot avg\_pm\_pen\_ni_0\,(L_1(x,y_1), L_2(x,y_2), \ldots, L_m(x,y_m)) + \sum_{i=1}^{m} c_i y_i - \sum_{j=1}^{n} \pi_j x_j\right\} \qquad \text{(CS.1)}$$

subject to

vehicle capacity

$$\sum_{i=1}^{m} c_i y_i \leq V \tag{CS.2}$$

bounds on decision variables

$$y_i \geq 0,\ i = 1, \dots, m; \qquad x_j \in \{0, 1\},\ j = 1, \dots, n. \tag{CS.4}$$