CASE STUDY: Portfolio Management with Basel Accord (var_dev, cvar_dev)

Background

This case study demonstrates an optimization setup for credit portfolio management. It is based on papers by Theiler, et al. (2003) and Theiler (2004). Similar optimization models for credit risk were considered in Andersson, et al. (2001). This model maximizes the expected returns of the credit portfolio under internal and regulatory loss risk limits. From the bank's internal perspective, credit risks are limited by the economic capital, i.e., the capital resources available to the bank to cover credit losses. The economic capital usually is defined as a subset of the bank's equity. At the same time, the bank needs to limit its credit risk from a regulatory perspective. We consider the loss risk limitation rules set by the Basel Committee on Banking Supervision. We are considering the prevailing rules of Basel I, Basel (1988, 1996). However, credit risk weights of the Basel II rules, Basel (2001), can be easily incorporated in similar way. Banks are charged capital to cover credit risks of their bank book which are limited by the maximum amount of regulatory capital applicable to cover these risks. We concentrate on a credit portfolio of the bank book. The credit risk of the bank book is limited by the "tier_1", i.e. the core capital, and the "tier_2", i.e. the supplementary capital. The tier_1 capital mainly consists of the core capital of the bank, plus some other components. The tier_2 capital includes supplementary capital elements, such as the allowance for loan loss reserves and various long-term debt instruments, such as subordinated debt, see, Basel (1988), and also United (1998), p. 119. This model integrates assets involving both market and credit risk under internal and regulatory loss risk limitations. The capital constraints limit the expected profits of the bank in the planning period. The less economic and regulatory capital are available, the less risk a bank is able to take, and the more limited the achievable expected profits are in a business period. We assume a planning horizon of one year for expected returns, one year for credit risk, and one day for market risk. We combine different horizons for credit and market risks under the assumption that portfolio positions are constant for the year and the market risk is the same (is constant) for every day of this year.

To provide background on risk-based regulations we extracted from United (1998) several relevant citations:

Credit risk

"Banks are required to meet a total risk-based capital requirement equal to 8 percent of risk-weighted assets. At a minimum, a bank's capital must consist of core capital, also called tier 1 capital, of at least 4 percent of risk-weighted assets. Core capital includes common stockholders' equity, noncumulative perpetual preferred stock, and minority equity investments in consolidated subsidiaries. The remainder of a bank's total capital can also consist of supplementary capital, known as tier 2 capital. This can include items such as general loan and lease loss allowances, cumulative preferred stock, certain hybrid (debt/equity) instruments, and subordinated debt with a maturity of 5 years or more. The regulation limits the amount of various items included in tier 1 and tier 2 capital. For example, the amount of supplementary (tier 2) capital that is recognized for purposes of the risk-based capital calculation cannot exceed 100 percent of tier 1 capital."

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"Under the credit risk rules, the adjustments of asset values to account for the relative riskiness of a counterparty involve multiplying the asset values by certain risk weights, which are percentages ranging from 0 to 100 percent. A zero risk-weight reflects little or no credit risk. For example, if a bank holds a claim on the U.S. Treasury, a Federal Reserve Bank, or the central government or central bank of another qualifying Organization for Economic Cooperation and Development

(OECD) country, this asset is multiplied by a factor of 0 percent, which results in no capital being required against the credit risk from this transaction."

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"For an obligation owed by another commercial bank in an OECD country, a bank must multiply the amount of this obligation by 20 percent, which has the effect of requiring the bank to hold capital equal to 1.6 percent of the value of the claim on the other bank. Loans fully secured by a mortgage on a 1-4 family residential property carry a risk weight of 50 percent, thus requiring the bank to hold capital equal to 4 percent of the value of the mortgage. For an unsecured obligation owed by a private corporation or individual, such as a loan without collateral, a bank must multiply the amount of the unsecured obligation by 100 percent, which requires the bank to hold capital equal to a full 8 percent of the value of the unsecured obligation."

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"To adjust for credit risks created by financial positions not reported on the balance sheet, the regulations provide conversion factors to express off-balance sheet items as an equivalent on-balance sheet item, as well as rules for incorporating the credit risk of interest-rate, exchange-rate, and other off-balance sheet derivatives. These positions are converted into a credit equivalent amount, and then the standard loan risk-weight for the type of customer is applied. The risk-weight is applied according to the type of obligor, except that in the case of derivatives the maximum risk-weight is 50 percent."

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"In both the banking and securities/futures sectors, capital regulations contain formulas that apply single risk-weightings to a broad range of riskiness within a single category. For example, in banking, the same 8 percent capital requirement is imposed on all unsecured loans to private commercial borrowers regardless of individual creditworthiness, with the result that a high-risk/high-return loan carries no more regulatory capital than a low-risk/low-return loan. As a result, the regulation might give firms an incentive to seek the highest returns within a broad class regardless of underlying risk; or to adjust activities (e.g, develop new products and/or change operations or corporate structures) in a way that reduces or escapes capital requirements. In other words, firms may adjust business to achieve the lowest regulatory capital cost rather than an optimal balance of risk and capital. Also, the securities net capital rule requires registered broker-dealers to apply a 100-percent haircut to any portion of the trading profits, to the extent the profits are unsecured, reflecting SEC's emphasis on liquidity in its net capital rule."

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"All banks are required to calculate their credit risk for assets, such as loans and securities; and off-balance sheet items, such as derivatives or letters of credit. The credit risk calculation assigns all assets and off-balance sheet items to one of four broad categories of relative riskiness (0, 20, 50, or 100 percent) according to type of borrower/obligor and, where relevant, the nature of any qualifying collateral or guarantee. Off-balance sheet items are converted into credit equivalent amounts. The assets and credit equivalent amount of off-balance sheet items in each category are multiplied by their appropriate risk-weight and then summed to obtain the total risk-weighted assets for the denominator of the credit risk-based capital ratio. Capital, the numerator of the capital ratio, is long-term funding sources for the bank that are specified in the regulations. A bank is to maintain a total risk-based capital ratio (total capital/risk-weighted assets) of at least 8 percent."

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"The credit risk regulation requires the use of two sets of multipliers. One set of multipliers places each off-balance sheet item into one of four categories and converts items in each category into asset equivalents. These conversion factors are multiplied by the face or notional amount of the off-balance sheet items to determine the "credit equivalent" amounts. In addition, for derivatives, these credit equivalent amounts are the value of the bank's claims on the

counterparties plus add-on factors to cover the potential future value of the derivative contracts. Then the other set of multipliers applies the risk-weights to assets and off-balance sheet credit equivalent amounts according to the type of borrower/obligor (and, where relevant, the nature of any qualifying collateral or guarantee). The sum of the risk-weighted assets in all categories is the credit risk-weighted assets for the bank."

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Market risk.

"Market risk consists of general market and specific risk components. To determine the market risk-equivalent assets, the risk or capital charges must be calculated for both components.

Market risk capital charges are based on general market and specific risks. Examples of general market risk factors are interest rate movements and other general price movements. Capital charges for general market risks are to be based on internal models developed by each bank to calculate a VAR estimate, i.e., potential loss that capital will need to absorb. The internal VAR estimate for general market risks is to be based on statistical analyses that determine the probability of a given loss, based on at least 1 year of historical data. This VAR estimate is to be calculated daily using a 99 percent one-tailed confidence interval with a price shock equivalent to a 10-business day movement in rates and prices; i.e., 99 percent of the time the calculated VAR would not be exceeded in a 10-day period."

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"Specific risk arises from factors relating to the characteristics of specific issuers of instruments. Specific risk factors reflect both idiosyncratic price movements of individual securities and "event risk" from incidents, such as defaults or credit downgrades, which are unique to the issuer and not related to market factors. If a bank's internal model does not capture all aspects of specific risk, an add-on to the capital charge is required for specific risk. Specific risk estimates based on internal models are subject to adjustments based on the precision of the model.

The total market risk capital charge is the sum of the capital charges for general market and specific risk. The total market risk capital charge is based on the larger of the previous day's VAR estimate and the average of the daily VAR estimates for the past 60 days times the multiplier. The multiplier ranges from 3 up to a maximum of 4 depending on the results of backtesting.17 Market risk-equivalent assets are the total market risk capital charges multiplied by 12.5."

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"Application of the market risk capital ratio requires the use of a two-part test. The sum of tiers 1, 2, and 3 capital must equal at least 8 percent of total adjusted risk-weighted assets. The tier 3 capital in this sum is only to be allocated to cover market risk. In addition, the sum of tier 2 and tier 3 capital for market risk may not exceed 250 percent of tier 1 capital allocated for market risk. The regulation includes other restrictions on the use of tier 2 and 3 capital."

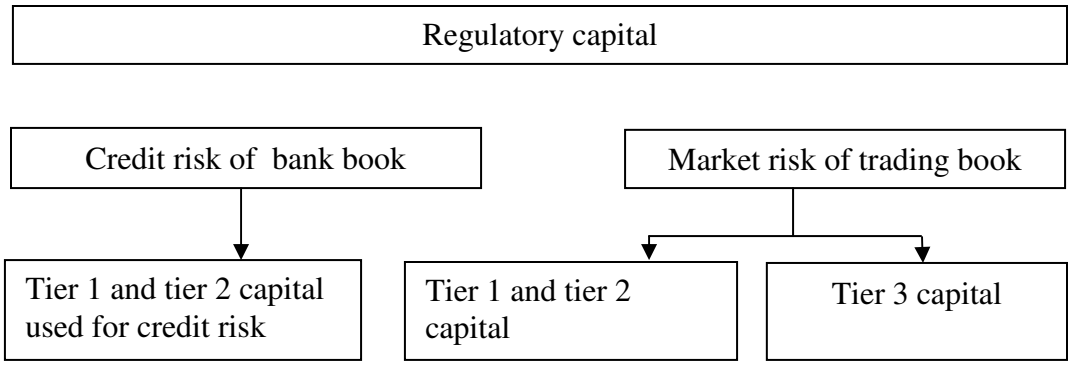

Figure 1. Regulatory Capital.

References

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- Basel committee on Banking Supervision (1988): International convergence of capital measurement and capital standards, Basel, July 1988.

- Basel committee on Banking Supervision committee (1996): Amendment to the capital accord to incorporate market risks, Basel, January 1996.

- Basel Committee on Banking Supervision (2001): Consultative Document. The New Basel Capital Accord, January 2001, Basel, January 2001.

- Theiler, U., Bugera, V., Revenko, A., and S. Uryasev (2003): Regulatory Impacts on Credit Portfolio Management. Leopold-Wildburger, U. et al. (Eds.), Operations Research Proceedings 2002, Springer, Berlin, 335-340.

- Theiler, U. (2004): Risk Return Management Approach for the Bank Portfolio in: Szego, G. (Ed.), Risk Measures for 21st Century, John Wiley & Sons, Chichester, 403-430.

- United States Accounting Office (1998): Risk-Based Capital - Regulatory and Industry Approaches to Capital and Risk, Washington, July 1998.

Several papers in this list can be downloaded from:
- http://www.ise.ufl.edu/uryasev/pubs.html#b
- http://www.ursula-theiler.de/publications.htm
- http://www.gloriamundi.org/

Notations

I = number of instruments (bonds) in the portfolio; i={1,…,I} index of instruments in the portfolio;

J = number of scenarios; j={1,…,J} index of scenarios;

$\mathbf{x} = (x_1, ..., x_I)$ = vector of exposures (in currency) to instruments i=1,…I ;

l_i = lower bound on exposure to instrument i ;

u_i = upper bound on exposure to instrument i ;

q_i = present value (price) of i-th instrument;

r_i = rate of return (per year) of i-th instrument in the absence of risk (for instance, yield of the bond);

θ_{ij}^{bb} = future value (in one year) of i-th instrument in the bank book under the credit risk scenario j accounting for credit migration and default;

$r_{ij}^{bb} = \dfrac{\theta_{ij}^{bb}}{q_i} - 1$ = rate of return (per year) of i-th instrument in the bank book under the credit risk scenario j accounting for credit migration and default.

$\mathbf{r}_j^{bb} = \left(r_{1j}^{bb}, ..., r_{Ij}^{bb}\right)$ = vector of rates of return (per year) of instrument i=1,…I in the bank book under the credit risk scenario j ;

$r_{ij}^{tb}(t)$ = rate of return (per 10 trading days) of i-th instrument in the trading book under the market risk scenario j ;

$\mathbf{r}_j^{tb} = \left(r_{1j}^{tb}, ..., r_{Ij}^{tb}\right)$ = vector of rates of return (per 10 trading) of instrument i=1,…I in the trading book under the credit risk scenario j ;

$L(\mathbf{x}, r_j^{bb}) = -\sum_{i=0}^{I} r_{ij}^{bb} x_i$ = bank book loss under the credit risk scenario j;

$L\left(\mathbf{x}, r_j^{tb}\right) = -\sum_{i=0}^{I} r_{ij}^{tb} x_i$ = trading book loss (per 10 trading days) under the market risk scenario j ;

α^{tb} = confidence level for VaR deviation for trading book;

α^{bb} = confidence level for CVaR deviation for bank book;

$\boldsymbol{C_{Tier-k}}$ = available Tier- k capital, k=1, 2,3;

$\boldsymbol{x_k^a}$ = used for risk management purposes Tier- k capital, k=1,…, 3 (free additional variables);

$\boldsymbol{w_i^{cr}}$ = regulatory credit risk capital weight for security *i;*

$\boldsymbol{w_i^{sp}}$ = regulatory specific market risk weight for security *i;*

$\boldsymbol{w^{mr}}$ = regulatory weight for market risk;

$\boldsymbol{C_{econ}}$ = maximum amount of economic capital available to cover internal loss risk (measured by CVaR deviation $\boldsymbol{CVaR\_DEV}\left(\boldsymbol{L}(\mathbf{x},\mathbf{r}^{bb})\right)$) .

Simulation of Scenarios

Yearly credit risk scenarios of bond returns, $\boldsymbol{r_{ij}^{bb}}$, accounting for credit migration and default can be simulated using standard methodologies, including CreditMetrics. 10-day market risk scenarios, $\boldsymbol{r_{ij}^{tb}}$, can be calculated with historical Monte Carlo simulations.

Optimization Problem

maximizing estimated return (without risk)

$$\max \sum_{i=1}^{I} r_i x_i \tag{CS.1}$$

subject to

internal constraint on credit risk

$$\boldsymbol{CVaR\_DEV}\left(\boldsymbol{L}(\mathbf{x},\mathbf{r}^{bb})\right) \leq \boldsymbol{C_{econ}} \tag{CS.2}$$

regulatory constraint on capital covering credit risk

$$\sum_{i=1}^{I} \boldsymbol{w_i^{cr}} \boldsymbol{x_i} = \boldsymbol{x_1^a} + \boldsymbol{x_2^a} \tag{CS.3}$$

regulatory constraint on capital covering market risk

$$\sum_{i=0}^{I} \boldsymbol{w_i^{sp}} \boldsymbol{x_i} + \boldsymbol{w^{mr}}\ \boldsymbol{VaR\_DEV}\left(\boldsymbol{L}(\mathbf{x},\mathbf{r}^{tb})\right) \leq \boldsymbol{x_3^a} + (\boldsymbol{C_{Tier-1}} - \boldsymbol{x_1^a}) + (\boldsymbol{C_{Tier-2}} - \boldsymbol{x_2^a}) \tag{CS.4}$$

constraint limiting unused Tier-2 + used Tier-3 capital vs. unused Tier-1 capital

$$x_3^a + (C_{Tier-2} - x_2^a) \leq 2.5\,(C_{Tier-1} - x_1^a) \qquad \text{(CS.5)}$$

constraint limiting Tier-2 vs. Tier-1 capital

$$x_2^a \leq x_1^a \qquad \text{(CS.6)}$$

upper/lower bounds on exposures

$$l_i \leq x_i \leq u_i, \quad i = 1,\ldots I; \qquad \text{(CS.7)}$$

bounds on used Tier- k capital

$$0 \leq x_k^a \leq C_{Tier-k}\,, \; k=1,\ldots,3\,. \qquad \text{(CS.8)}$$

Comment

According to the Basel accord, see, United (1998), "The total market risk capital charge is based on the larger of the previous day's VaR estimate and the average of the daily VaR estimates for the past 60 days" of the minimal return over 10 trading days. As a proxy for this VaR estimate, we considered in the model the VaR estimate of 10 trading days returns. This is an optimistic estimate of the value which should be included in the model. After solving the optimization problem the actual risk constraints can be verified for the optimal portfolio. If the actual VaR constraint included in regulations is not satisfied, then the coefficient w^{mr} can be increased and the optimization problem can be solved with a higher weight for the market risk.