Optimal Crop Planting Schedule and Hedging Strategy Under ENSO-based Climate Forecast

Chung-Jui Wang[1], Victor E. Cabrera[2], Stan Uryasev[3], and Clyde W. Fraisse[4]

RESEARCH REPORT #2007-2

Risk Management and Financial Engineering Lab
Department of Industrial and Systems Engineering
University of Florida, Gainesville, FL 32611

Correspondence should be addressed to: Stan Uryasev

Abstract

This article investigates the optimal crop planting schedule and hedging strategy in the mean return versus CVaR risk framework. Crop insurances and futures contracts are available for hedging against yield and price risks. The impact of the ENSO-based climate forecast on the optimal production and hedging decision is examined. Gaussian copula is applied in simulating the scenarios of correlated non-normal random yields and prices. Using data of a representative cotton producer in the Southeastern United States, the best production and hedging strategy is evaluated under various risk tolerances for each ENSO phases.

Keywords: planting schedule, crop insurance, futures contract, Conditional Value-at-Risk (CVaR), El Niño Southern Oscillation (ENSO), simulation, copula

[1] Risk Management and Financial Engineering Lab., Department of Industrial and Systems Engineering, University of Florida; Email: cjwang@ufl.edu; URL: plaza.ufl.edu/cjwang
[2] Dairy Extension, College of Agricultural and Home Economics, New Mexico State University; Email: victorc@njcf.ad.nmsu.edu
[3] Risk Management and Financial Engineering Lab., Department of Industrial and Systems Engineering, University of Florida; Email: uryasev@ufl.edu; URL: www.ise.ufl.edu/uryasev
[4] Department of Agricultural and Biological Engineering, University of Florida; Email: cfraisse@ufl.edu

A risk-averse farmer who wants to maximize the profit from growing crops faces uncertainty in the crop yields and harvest price. To manage uncertainty, a farmer may purchase a crop insurance product and/or trade futures contracts against the yield and price risks. Crop yields depend on planting dates and weather conditions during the growing period. The predictability of seasonal climate variability (i.e., the El Niño Southern Oscillation, ENSO) gives the opportunity to forecast crop yields in different planting dates. Furthermore, the flexibility in planting timing allows a farmer to optimize profit by selecting a best planting schedule and a hedging strategy according to ENSO forecast.

Two common financial instruments for hedging against crop risks are crop insurances and futures contracts. The Risk Management Agency (RMA) of the United States Department of Agricultural (USDA) offers crop insurance policies for various crops, which are categorized into three types: the yield-based insurance, revenue-based insurance, and policy endorsement. The yield-based insurance policy, e.g., Actual Production History (APH), insures producers against yield losses due to natural causes. The revenue-based insurance policy, such as Crop Revenue Coverage (CRC), provides revenue protection. Catastrophic Coverage (CAT), a policy endorsement, pays 55% of the price, established annually by RMA, of the commodity on crop yield shortfall in excess of 50%. The cost of crop insurances includes a premium and an administration fee. In addition to crop insurance coverage, farmers may manage commodity price risk by futures contracts, an agreement between two parties to buy or sell a a specific amount of a commodity at a certain time in the future at a certain price. Futures contracts are highly standardized and are traded by exchange. The cost of futures contract includes commissions and interest foregone on margin deposit. A risk-averse producer may consider using insurance products in conjunction with futures contacts for the best possible outcome.

El Niño Southern Oscillation refers to interrelated atmospheric and oceanic phenomena. The barometric pressure difference between the eastern and western equatorial Pacific frequently changes. This phenomenon is known as the Southern Oscillation. El Niño refers to the rise in the sea surface temperature (SST) due to the western Pacific being above and the eastern Pacific pressure being below normal. On the other hand, when the east-west barometric pressure gradient is reversed, SST drops below normal. This is called La Niña. SSTs within a normal temperature range are called “neutral”. These equatorial Pacific conditions, known as ENSO phases, refer to different seasonal climatic conditions. Since the Pacific SSTs are predictable, ENSO has become an index for forecasting climate and, consequently, crop yields.

A great deal of research has focused on crop risk hedging using crop insurance and other derivative securities. Poitras (1993) studied farmers’ optimal hedging problem when both futures and crop insurance are available to reduce the uncertainty of price and production. Chambers and Quiggin (2002) examined optimal producer behavior in the presence of area-yield insurance. Mahul (2003) investigated the demand of futures and options for hedging against price risk when crop yield and revenue insurance contracts are available. Coble, Miller, and Zuniga (2004) investigated the effect of crop insurance and loan programs on demand for futures contracts.

Cane, Eshel, and Buckland (1994), Hansen, Hodges, and Jones (1998), Hansen (2002), and Jones et al. (2000) have investigated the connection between the ENSO-based climate prediction and crop yields. Recently, some researchers have studied the impacts of the ENSO-based climate information on the selection of optimal crop insurance policies. Cabrera et al. (2006) examined the impact of ENSO-based climate forecast on reducing farm risk with optimal crop insurance strategy. Cabrera, Letson, and Podesta (2007) included the interference of farm government programs on crop insurance hedge under ENSO climate forecast.

The purpose of this article is two-fold. First, we propose a mean - CVaR optimization model to investigate the optimal crop planting schedule and hedging strategy when crop insurances and futures contracts are available to the producer. Second, we examine the impact of the ENSO-based climate forecast on the optimal decisions of crop planting schedule and hedging strategy. Also, yield and price are not typically normally distributed and the potential negative correlation between random production and price provides a crucial natural hedge (McKinnon 1967) in risk management. Therefore, we consider the correlation and non-normality features of yield and price for crop planting schedule and hedging strategy analysis, and applied Gaussian copula to model the dependent and non-normal distributions of crop yields and price.

The remaining article is organized as follows. First, the optimization model is introduced. Then an empirical case study using the data of a representative cotton producer in the Southeastern United States is described. It is followed by the results and discussions of the optimal planting schedule and hedging strategy for each ENSO phase. The final section presents the conclusions.

Model

To investigate the impact of ENSO-based climate forecast on the optimal production and risk management decisions, we calibrate the joint distribution of crop yields and price for individual ENSO phases based on the historical yields and prices of the years classified to the ENSO phase. Then, random yield and price scenarios associated with the ENSO phase are generated by Monte Carlo simulation.

Random yield and price simulation with copula

We assume a farmer may plant crops in a number of planting dates across the planting season. The correlations between yields in various planting dates and between yields and price are considered and modeled by copula method. Copulas are functions describing dependencies among variables, and providing a way to create distributions to model correlated multivariate data. The Sklar theorem (Sklar 1959) states that given a joint distribution function F on R^n with marginal distribution F_i, there is a copula function C such that for all $x_1,...,x_n$ in R,

(1) $$F(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)).$$

Furthermore, if F_i are continuous then C is unique. Conversely, if C is a copula and F_i are distribution functions, then F as defined by the previous expression is a joint distribution function with margins F_i. We apply the Gaussian copula method to generate the correlated non-normal multivariate distribution. The Gaussian copula is given by:

(2) $$C_\rho(F_1(x_1),...,F_n(x_n)) = \Phi_{n,\rho}(\Phi^{-1}(F_1(x_1)),...,\Phi^{-1}(F_n(x_n))),$$

which maps the observed variable x_i, i.e. yield or price, into a new variable y_i using the transformation

(3) $$y_i = \Phi^{-1}\left[F_i(x_i)\right],$$

where $\Phi_{n,\rho}$ is the joint distribution function of a multivariate Gaussian vector with a mean of zero and correlation matrix ρ. Φ is the distribution function of a standard Gaussian random variable. In moving from x_i to y_i the observation from the assumed distribution F_i is mapped into a standard normal distribution Φ on a percentile to percentile basis.

We use the rank correlation coefficient Spearman's rho, ρ_s, to calibrate the Gaussian copula to the historical data. For *n* pairs of bivariate random samples $\left(X_i, X_j\right)$, define $R_i = rank(X_i)$ and $R_j = rank(X_j)$. Spearman's sample rho is given by

(4) $$\rho_s = 1 - 6\frac{\sum_{i=1}^{n}\left(R_i - R_j\right)}{n(n^2-1)}.$$

Spearman's rho measures the association only in terms of ranks. The rank correlation is preserved under the monotonic transformation in equation 3. Furthermore, there is a one-to-one mapping between rank correlation coefficient Spearman's rho, ρ_s, and linear correlation coefficient, ρ, for the bivariate normal random variables (y_1, y_2) (Kruksal 1958)

(5) $$\rho_s(y_1, y_2) = \frac{6}{\pi}\arcsin\frac{\rho(y_1, y_2)}{2}.$$

To generate correlated multivariate non-normal random variables with margins F_i and Spearman's rank correlation ρ_s, we generate the random variables y_i's from the multivariate normal distribution $\Phi_{n,\rho}$ with linear correlation

(6) $$\rho = 2\sin\left(\frac{\pi\rho_s}{6}\right),$$

by Monte Carlo simulation. The actual outcomes x_i 's can be mapped from y_i 's using the transformation

$$x_i = F_i^{-1}\left[\Phi(y_i)\right]. \tag{7}$$

Mean-CVaR model

Since Markowitz (1952) proposed the mean-variance framework in portfolio optimization, variance/covariance has become the predominant risk measure in finance. However, this risk measure is suited only to elliptic distributions, like normal or t-distributions, with finite variances (Szegö 2002). The other drawback of variance risk measure is that it measures both upside and downside risks. In practice, finance risk management is concerned mostly with the downside risk. A popular downside risk measure in economics and finance is Value-at-Risk (VaR) (Jorion 2000), which measures α percentile of loss distribution. However, as was shown by Artzner et al. (1999), VaR is ill-behaved and non-convex for general distribution. The disadvantage of VaR is that it only considers risk at α percentile of loss distribution and does not consider the magnitude of the losses in the α-tail (the worst 1-α percentage of scenarios).

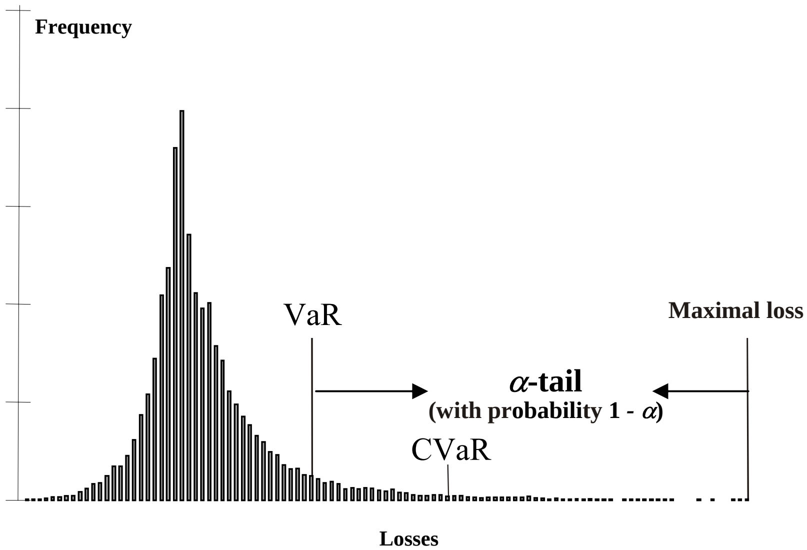

Figure 1. The definition of VaR and CVaR associated with a loss distribution

showed that CVaR constraints in optimization problems can be formulated as a set of linear constraints and incorporated into the problems of optimization. This linear property is crucial to formulate the model as a mixed 0-1 linear programming problem that can be efficiently solved.

This article proposes a mean-CVaR model, like mean-variance model, provides an efficient frontier consisting of points that maximize expected return under various tolerances of CVaR losses. Since CVaR is defined in monetary units, farmers may decide their risk tolerance much more intuitively compared to abstract utility functions. It is worth noting that CVaR is defined on a loss distribution, so a negative CVaR value represents a profit.

Model implementation

We consider a farmer who plans to grow crops in a farmland of Q acres. There are K possible types of crops and more than one crop can be planted at a time. For each crop k, there are T_k potential planting dates that give different yield distributions based on the predicted ENSO phase, as well as I_k available insurance policies for the crop. The decision variables x_{kti} represents the acreages of crop k planted in date t with insurance policy i and η_k represents the hedge position (in pounds) of crop k in a futures contract.

The randomness of crop yield and harvest price in a specific ENSO phase is managed by the joint distribution corresponding to the ENSO phase. We sample J scenarios from the joint distribution by Monte Carlo simulation with a Gaussian copula, with each scenario having equal probability. Let Y_{ktj} denote the j^{th} realized yield (pound per acre) of crop k planted on date t, and P_{kj} denote the j^{th} realized cash price (dollar per pound) for crop k at the time the crop will be sold.

The objective function of the model, shown in equation (8), is to maximize the expectation of random profit $f(x_{kti}, \eta_k)$ that consists of the random profit from production $f^P(x_{kti})$, crop insurance $f^I(x_{kti})$, and futures contract $f^F(\eta_k)$.

$$\max Ef(x_{kti}, \eta_k) = \max\left[Ef^P(x_{kti}) + Ef^I(x_{kti}) + Ef^F(\eta_k)\right]. \tag{8}$$

The profit from production of crop k in scenario j is equal to the income from selling the crop, $\sum_{t=1}^{T_k}\left(Y_{ktj}P_{kj}\sum_{i=1}^{I_k}x_{kti}\right)$, minus the production cost, $C_k\sum_{t=1}^{T_k}\sum_{i=1}^{I_k}x_{kti}$, plus the subsidy, $S_k\sum_{t=1}^{T_k}\sum_{i=1}^{I_k}x_{kti}$, where C_k and S_k are unit production cost and subsidy, respectively. Consequently, equation (9) expresses the expected profit from production.

(9) $$Ef^{P}(x_{kti}) = \frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T_k}\left(Y_{ktj}P_{kj} - C_k + S_k\right)\sum_{i=1}^{I_k} x_{kti}\ .$$

Three types of crop insurance policies are considered in the model, including Actual Production History (APH), Crop Revenue Coverage (CRC), and Catastrophic Coverage (CAT). For APH, farmers select the insured yield, a percentage α_i from 50 to 75 percent with five percent increments of average yield $\overline{Y}_k$, as well as the election price, a percentage, β_i , between 55 and 100 percent of the of the established price P_k established annually by RMA. If the harvest is less than the yield insured, the farmer is paid an indemnity based on the difference $\sum_{t=1}^{T_k}\left(\alpha_i\overline{Y}_k - Y_{ktj}\right)x_{kti}$ at price $\beta_i P_k$. The indemnity of APH insurance policy $i \in I_{APH}$ for crop *k* in the j^{th} scenario is shown in equation (10).

(10) $$D_{kij} = \max\left[\sum_{t=1}^{T_k}\left(\alpha_i\overline{Y}_k - Y_{ktj}\right)x_{kti}, 0\right] \times \beta_i P_k \qquad \forall i \in I_{APH}$$

For CRC, producers select a percentage of coverage level γ_i between 50 and 75 percent. The guaranteed revenue is equal to the coverage level γ_i times the product of $\sum_{t=1}^{T_k}\overline{Y}_k x_{kti}$ and the higher of the base price (early-season price) P_k^b and the realized harvest price in the j^{th} scenario of crop *k*, P_{kj}^h. The base price and harvest price of crop *k* are generally defined based on the crop's futures price in planting and harvest seasons respectively. If the calculated revenue $\sum_{t=1}^{T_k} Y_{ktj}x_{kti}P_{kj}$ is less than the guaranteed one, the insured farmers will be paid the difference. Equation (11) shows the indemnity of a CRC insurance policy $i \in I_{CRC}$ for crop *k* in the j^{th} scenario.

$$D_{kij} = \max\left[\gamma_i \sum_{t=1}^{T_k} \overline{Y}_k x_{kti} \times \max\left[P_k^b, P_{kj}^h\right] - \sum_{t=1}^{T_k} Y_{ktj} x_{kti} P_{kj}, 0\right] \qquad \forall i \in I_{CRC}\,. \tag{11}$$

The CAT insurance pays 55 % of the established price of the commodity on crop losses in excess of 50 %. The indemnity of CAT insurance policy $i \in I_{CAT}$ for crop *k* in the j^{th} scenario is shown in equation (12).

$$D_{kij} = \max\left[\sum_{t=1}^{T_k}\left(0.5\overline{Y}_k - Y_{ktj}\right)x_{kti}, 0\right] \times 0.55 P_k \;. \tag{12}$$

The cost of insurance policy *i* for crop *k* is denoted by R_{ki}, which includes a premium and an administration fee. For the case of CAT, the premium is paid by the Federal Government. Therefore, the cost of CAT is only a $ 100 administrative fee for each crop insured in each county.

The expected total profit from insurance is equal to the indemnity from the insurance coverage minus the cost of the insurance that is shown in equation (13)

$$Ef^{I}\left(x_{kti}\right) = \left\{\frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{i=1}^{I_k} D_{kij} - R_{ki}\sum_{t=1}^{T_k} x_{kti}\;. \tag{13}$$

The payoff of a futures contract for crop *k* in scenario *j* for a seller is shown below

$$\pi_{kj}^{F} = (F_k - f_{kj})\eta_k\,, \tag{14}$$

where F_k is the futures price of crop *k* in the planting time, f_{kj} is the j^{th} realized futures price of crop *k* in the harvest time, and η_k is the hedge position (in pounds) of crop *k* in futures contract. It is worth noting that the futures price f_{kj} is not exactly the same as the local cash price P_{kj} at harvest time. Basis, defined in equation (15), refers to the difference that induces a type of uncertainty of futures hedging known as the basis risk. The random basis can be estimated from comparing the historical cash prices and futures prices.

(15) Basis = Cash Price – Futures Price.

The cost of a futures contract, C_k^F, includes commissions and interest foregone on margin deposit. Equation (16) expresses the expected profit from futures contract.

$$Ef^F(\eta_k) = \frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{K}\left(\pi_{kj}^F - C_k^F\right) \quad (16)$$

We introduce binary variables z_{ki} in constraints (17) and (18) to ensure that only one insurance policy can be selected for each crop *k*.

$$\sum_{t=1}^{T_k} x_{kti} \le Q \cdot z_{ki} \qquad \forall i, k \quad (17)$$

$$\sum_{i=1}^{I_k} z_{ki} = 1 \qquad \forall k \quad (18)$$

where

$$z_{ki} = \begin{cases} 1 & \text{if crop } k \text{ is insured by policy } i, \\ 0 & \text{otherwise.} \end{cases}$$

Constraint (19) restricts the total planting area to a given planting acreage *Q*. The equality in this constraint can be replaced by an inequality $(\le)$ to represent a farmer's choice to not grow the crops when the production is not profitable.

$$\sum_{k=1}^{K}\sum_{t=1}^{T_k}\sum_{i=1}^{I_k} x_{kti} = Q \quad (19)$$

To model producer's risk tolerance, we impose the CVaR constraint

$$CVaR_\alpha(L(x_{kti}, \eta_k)) \le U\,. \quad (20)$$

where $L(x_{kti}, \eta_k)$ is a random loss equal to the negative random profit $f(x_{kti}, \eta_k)$ defined in (8). The definition of $CVaR_\alpha(L(x_{kti}, \eta_k))$ is shown in equation (21),

$$CVaR_\alpha(L(x_{kti}, \eta_k)) = E\left[L(x_{kti}, \eta_k) \middle| L(x_{kti}, \eta_k) \ge \zeta_\alpha\left(L(x_{kti}, \eta_k)\right)\right], \quad (21)$$

where $\zeta_\alpha(L(x_{kti},\eta_k))$ is the α-quintile of the distribution of $L(x_{kti},\eta_k)$. Therefore, constraint (20) ensures that the conditional expectation of the random loss $L(x_{kti},\eta_k)$, given that the random loss exceeds α-quintile, is less than or equal to *U*. In other words, the expected loss of α-tail, i.e. (1- α)100% worst scenarios, is upper limited by an acceptable CVaR upper bound *U*. Rockafellar & Uryasev (2000) showed that *CVaR* constraint (20) in optimization problems can be expressed by linear constraints (22), (23), and (24)

(22) $$\zeta_\alpha(L(x_{kti},\eta_k))+\frac{1}{J(1-\alpha)}\sum_{j=1}^{J} z_j \le U,$$

(23) $$z_j \ge \sum_{k=1}^{K}\sum_{t=1}^{T_k}\sum_{i=1}^{I_k} L(x_{kti},\eta_k)-\zeta_\alpha(L(x_{kti},\eta_k)) \qquad \forall j,$$

(24) $$z_j \ge 0 \qquad \forall j,$$

where z_j are artificial variables introduced for the linear formulation of CVaR constraint. Note that the maximum objective function contains indemnities D_{kij} that include a *max* term shown in equations (10), (11), and (12). To implement the model as a mix 0-1 linear problem, we transform the equations to an equivalent linear formulation by disjunctive constraints (Nemhauser and Wolsey, 1999). For example, equation (10), $D_{kij}=\max\left[\sum_{t=1}^{T_k}(\alpha_i\bar{Y}_k-Y_{ktj})x_{kti},0\right]\times\beta_i P_k$, can be represented by a set of mix 0-1 linear constraints

(25) $$D_{kij}\ge 0,$$
$$D_{kij}\ge\left[\sum_{t=1}^{T_k}(\alpha_i\bar{Y}_k-Y_{ktj})x_{kti}\right]\times\beta_i P_k,$$
$$D_{kij}\le\left[\sum_{t=1}^{T_k}(\alpha_i\bar{Y}_k-Y_{ktj})x_{kti}\right]\times\beta_i P_k+MZ_{kij},$$

$$D_{kij} \le \left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k + M(1\text{-}Z_{kij}) \ ,$$

$$\left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k \le M(1\text{-}Z_{kij}),$$

$$\left[\sum_{t=1}^{T_k}\left(\alpha_i \overline{Y}_k - Y_{ktj}\right)x_{kti}\right] \times \beta_i P_k \ge -MZ_{kij}.$$

where M is a large number and Z_{kij} is a 0-1 variable. Similarly, equations (11) and (12) can be transformed into a set of mix 0-1 linear constraints in the same way. Consequently, the optimal crop production and hedging problem can be formulated as a mix 0-1 linear programming problem.

Problem solving and decomposition

Although the mix 0-1 linear programming problem can be solved with optimization software, the solving time increases exponentially as the problem becomes large. To improve the solving efficiency, the original problem was decomposed into sub-problems in which each crop is insured by a specific insurance policy. The original problem contains K types of crops, and for the k^{th} type of crop there are I_k eligible insurance policies. Therefore, the number of the sub-problems is equal to the number of all possible insurance combinations of the K crops, $\prod_{k=1}^{K} I_k$. The sub-problems are the same as the original problem except that the index *i's* are fixed and equations (17) and (18) are removed. Solving the sub-problems gives the optimal production strategy and futures hedge amount under a specific combination of insurance policies for K crops. The solution of the sub-problem with the highest optimal expected profit gives the optimal solution of the original problem. The optimal production strategy and futures hedge position are provided from the said sub-problem solution and the optimal insurance coverage is the specific insurance combination of the sub-problem.

Case Study

The case study, following Cabrera et al. (2006), considers a representative farmer grows cotton in a non-irrigated farm of 100 acres in Jackson County, Florida. Dothan Loamy Sand, the dominant soil type in the region, is assumed. The farmer may buy futures contracts from the New York Board of Trade and/or purchase crop insurance against the crop yield and price risks. Three types of crop insurances, Actual Production History (APH), Crop Revenue Coverage (CRC), and Catastrophic Coverage (CAT), are available for cotton. The farmer may select only one insurance policy or opt for no insurance at all. For APH, the eligible coverage levels of yield are from 65% to 75% with 5% increments, and the election price is assumed to be 100% of the established price. In addition, the available coverage levels of revenue for CRC are from 65% to 85% with 5% increments.

Historical climate data from 1960 to 2003 is selected for the numerical implementation. ENSO phases during this period included 11 years of El Niño, 9 years of La Niña, and the remaining 24 years of Neutral, according to the Japan Meteorological Agency's definitions (Japan Meteorological Agency 1991).

Table 1. Historical Years Associated with ENSO Phases, 1960-2003

EL Niño		Neutral				La Niña	
1964	1987	1960	1975	1984	1994	1965	1989
1966	1988	1961	1978	1985	1995	1968	1999
1970	1992	1962	1979	1986	1996	1971	2000
1973	1998	1963	1980	1990	1997	1972	
1977	2003	1967	1981	1991	2001	1974	
1983		1969	1982	1993	2002	1976	

The cotton yields during the period of 1960-2003 were simulated using the CROPGRO-Cotton model (Messina, Jones, and Fraisse 2005) in the Decision Support System for Agrotechnology Transfer (DSSAT) v4.0 (Jones et al., 2003) based on the historical climate data collected at Chipley weather station. The input for the simulation model followed the current management practices of variety, fertilization and planting dates in the region. More specifically, a medium to full season Delta & Pine Land® variety (DP55), 110 kg/ha Nitrogen fertilization in two applications, and four planting dates, 16 Apr, 23 Apr, 1 May, and 8 May, were included in the yield simulation, which was further stochastically resampled to produce a series of synthetically generated yields following the historical distributions (for more details see Cabrera et al. 2006).

It was assumed that the cotton would be harvested and sold in December. Therefore, the December cotton futures contract was used to hedge the price risk. The historical settlement prices of the December futures contract on the last trading date from 1960 to 2003 were collected from the New York Board of Trade.

Table 2. Marginal Distributions and Rank Correlation Coefficient Matrix of Yields of Four Planting Dates and Futures Price for the Three ENSO Phases

ENSO	Variable	Statistics of Marginal Distribution		Rank Correlation Coefficient Matrix Spearman's rho				
		Mean	Standard Deviation	Yield (4/16)	Yield (4/23)	Yield (5/1)	Yield (5/8)	Futures Price
El Niño	Yield on 4/16	815.0	71.7	1.00	0.93	0.75	0.74	-0.36
	Yield on 4/23	804.6	79.4	0.93	1.00	0.63	0.57	-0.23
	Yield on 5/1	795.4	99.8	0.75	0.63	1.00	0.75	-0.22
	Yield on 5/8	793.7	79.1	0.74	0.57	0.75	1.00	-0.42
	Futures Price	0.5433	0.1984	-0.36	-0.23	-0.22	-0.42	1.00
Neutral	Yield on 4/16	808.9	108.8	1.00	0.84	0.77	0.62	-0.16
	Yield on 4/23	818.4	100.6	0.84	1.00	0.75	0.64	-0.28
	Yield on 5/1	825.8	86.2	0.77	0.75	1.00	0.75	-0.01
	Yield on 5/8	824.5	68.0	0.62	0.64	0.75	1.00	-0.19
	Futures Price	0.5699	0.1872	-0.16	-0.28	-0.01	-0.19	1.00
La Niña	Yield on 4/16	799.1	99.8	1.00	0.97	0.67	0.60	0.13
	Yield on 4/23	790.7	85.3	0.97	1.00	0.73	0.68	0.20
	Yield on 5/1	793.9	90.6	0.67	0.73	1.00	0.97	-0.13
	Yield on 5/8	809.3	94.1	0.60	0.68	0.97	1.00	-0.08
	Futures Price	0.4669	0.1851	0.13	0.20	-0.13	-0.08	1.00

The statistics and rank correlation coefficient Spearman's rho matrix of yields and futures price are summarized in table 2, which shows that crop yields for different planting dates are highly correlated and the correlation of yields decreases when the two corresponding planting dates are farther apart. In addition, the negative correlation between yields and futures price is found in the El Niño and Neutral phases, but not in La Niña. The random yields and futures price are assumed to follow the empirical distributions of yields and futures price.

We further estimated the local basis defined in equation (15). The monthly historical data on average cotton prices received by Florida farmers from the USDA National Agricultural Statistical Service (1979 to 2003) were assumed to be the cotton local cash prices. By subtracting the futures price from the local cash price, we estimated the historical local basis. Using the Input Analyzer in the simulation software Arena, it was found that the best fitted distribution based on minimum square error method was a beta distribution with probability density function, $-0.13+0.15\times$ BETA (2.76, 2.38).

The Gaussian copula was calibrated based on the sample rank correlation coefficient, Spearman's rho, matrix for the three ENSO phases. For each ENSO phase, 2,000 scenarios of correlated random yields and futures price were sampled based on the Gaussian copula and the empirical distributions of yields and futures price by Monte Carlo simulation. Further, the basis was simulated and the local cash price was calculated from the futures price and basis.

The futures commission and opportunity cost of margin was assumed to be $0.003 per pound, the production cost of cotton was $464 per acre, and the subsidy for cotton in Florida was $349 per acre. Finally, the parameters of crop insurance are listed in table 3.

Table 3. Parameters of Crop Insurance (2004) Used in the Farm Model Analysis

Crop Insurance Parameters	Values
APH premium 65%~75%	$19.5/acre ~$38/acre
CRC premium 65%~85%	$24.8/acre~$116.9/acre
Established Price for APH	$0.61/lb
Average yield	814 lb/acre

Source: www.rma.usda.gov

Results and Discussion

This section reports the results of optimal planting schedule and hedging strategy for the three predicted ENSO phases. First, the optimal decision when the crop insurances are the only hedging instrument was investigated. Then, the best solution when both insurances and futures contracts are available was analyzed.

Optimal production with crop insurance coverage

Table 4. Optimal Insurance and Production Strategies for Each Climate Scenario Under Various 90% CVaR Tolerance

ENSO Phases	90%CVaR Upper Bound	Optimal Expected Profit	Optimal Insurance Strategy	Optimal Planting Schedule			
				Date1	Date2	Date3	Date4
El Niño	<-18000	infeasible					
	-16000	28364	CRC70%	100	0	0	0
	-14000 to -4000	28577	CRC65%	100	0	0	0
	>-2000	28691	No	100	0	0	0
Neutral	<-20000	infeasible					
	-18000	31149	CRC70%	0	0	100	0
	-16000 to -10000	31240	CRC65%	0	0	100	0
	-8000	31779	No	0	0	85	15
	>-6000	31793	No	0	0	100	0
La Niña	<-12000	infeasible					
	-10000 to -6000	20813	CRC65%	100	0	0	0
	>-4000	21572	No	0	0	0	100

Note: Negative CVaR upper bounds represent profits. Four planting dates: 'Date1' = April 16, 'Date2' = April 23, 'Date3' = May 1, 'Date4' = May 8. "No" stands for no insurance.

Table 4 shows the optimal planting schedule and hedging strategy considering crop insurance as the only hedging instrument for each ENSO phase with various 90%CVaR upper bounds ranged from -$20,000 to -$2,000. Since the indemnity of CRC depends on the futures price, it is assumed that the futures market is unbiased, i.e., $F = Ef$ where F is the futures price in the planting time and f is the random futures price in the harvest time.

The remarks in table 4 are summarized as follows. First, the ENSO phases affected the expected profit and the feasible region of the downside risk. The Neutral year has the highest expected profit and lowest downside loss. In contrast, the La Niña year has the lowest expected profit and highest downside loss. Second, when the upper bound of 90%CVaR constraint is lower than a specific value, which depends on the ENSO phase, the 65%CRC and 70%CRC crop insurance policies are desirable for the optimal hedging strategy in all ENSO phases. In contrast, the APH insurance policies are not desirable for all ENSO phases and 90%CVaR upper bounds. Third, the risk can be managed by changing the planting schedule. The last two rows associated with the Neutral phase show that planting 100 acres on date 3 provides a 90%CVaR of -$6,000, which can be reduced to -$8,000 by planting 85 acres on date 3 and 15 acres on date 4. Last, changing both the insurance coverage and the planting schedule may reduce the downside risk. In the La Niña phase, planting 100 acres on date 4 provides a 90%CVaR of -$4,000 that can be reduced to -$10,000 by purchasing a 65%CRC insurance policy and shifting the planting date from date 4 to date 1.

Hedging with crop insurance and futures contract

The hedge ratio of the futures contract is defined as the hedge position in the futures contract divided by the expected production. We first illustrate the optimal hedging strategies and the optimal planting schedule. Then, the performance of the optimal hedge

Table 5. Optimal Insurance Policy and Futures Hedge Ratio Under Biased Futures Prices

Bias		-10%		-5%		0%		5%		10%	
ENSO Phases	90% CVaR Upper Bound	Ins.	H.R	Ins.	H.R	Ins.	H.R	Ins.	H.R	Ins.	H.R
El Niño	-28000							x		x	
	-24000			x		x		No	1.00	No	1.24
	-20000	x		65CRC	0.48	No	0.70	No	1.33	No	1.57
	-16000	70CRC	0.20	65CRC	0.15	No	0.52	No	1.60	No	1.87
	-12000	65CRC	-0.01	65CRC	-0.07	No	0.35	No	1.86	No	2.16
	-8000	65CRC	-0.20	65CRC	-0.24	No	0.19	No	2.11	No	2.44
	-4000	65CRC	-0.39	65CRC	-0.41	No	0.02	No	2.35	No	2.73
	0	65CRC	-0.58	65CRC	-0.58	No	0.00	No	2.60	No	3.01
Neutral	-30000									x	
	-28000							x		No	1.06
	-24000			x		x		No	1.19	No	1.42
	-20000	x		No	0.66	No	0.60	No	1.46	No	1.68
	-18000	70CRC	0.28	70CRC	0.12	No	0.50	No	1.57	No	1.80
	-16000	70CRC	0.10	70CRC	-0.02	No	0.40	No	1.68	No	1.92
	-14000	70CRC	-0.04	70CRC	-0.12	No	0.30	No	1.79	No	2.04
	-12000	70CRC	-0.16	No	0.20	No	0.20	No	1.89	No	2.15
	-8000	70CRC	-0.39	No	0.00	No	0.01	No	2.10	No	2.38
	-4000	70CRC	-0.62	No	-0.21	No	0.00	No	2.30	No	2.59
	0	70CRC	-0.86	No	-0.42	No	0.00	No	2.49	No	2.81
La Niña	-20000							x		x	
	-16000			x		x		No	1.04	No	1.23
	-12000	x		No	0.67	No	0.55	No	1.33	No	1.51
	-8000	No	0.33	No	0.27	No	0.23	No	1.57	No	1.77
	-4000	No	-0.10	No	-0.08	No	0.00	No	1.80	No	2.02
	0	No	-0.49	No	-0.41	No	0.00	No	2.02	No	2.27

Note: Negative CVaR upper bounds represent profits. “Ins.” stands for Insurance coverage. “H.R.” stands for hedge ratio. “No” stands for no insurance. “x” represents infeasible.

and planting strategies is introduced by the efficient frontiers on the expected profit versus CVaR risk diagram.

Table 5 shows the optimal insurance policy and futures hedge ratio associated with different 90%CVaR upper bounds for the three ENSO phases. When the futures price is unbiased or positive biased, the futures contract is the only desirable instrument for crop risk management and no insurance policy is needed in the optimal hedging strategy. On the other hand, when the futures price is negative biased, the optimal hedging strategy includes 65%CRC (or 70%CRC in some cases) insurance policies and futures contract in the El Niño phase for all feasible 90%CVaR upper bounds. In the Neutral phase, the optimal hedging strategy consists of the 70%CRC insurance policy and futures contract for all CVaR upper bounds with the -10% biased futures price and for CVaR upper bounds between -18000 and -14000 with -5% biased futures price. In addition, no insurance policy is desirable under the La Niña phase.

Mahul (2003) showed that the hedge ratio contains two parts: a pure hedge component and a speculative component. The pure hedge component corresponds to the hedge ratio associated with unbiased futures price. A positive biased futures price causes the farmer to select a long speculative position and a negative biased futures price implies a short speculative position. Therefore, the optimal futures hedge ratio under positive (negative) biased futures prices should be higher (lower) than that under the unbiased futures price. However, the optimal hedge ratios in table 5 do not agree with the conclusion when the futures price is negative biased. The optimal hedge ratios in the La Niña phase illustrate how optimal futures hedge ratios change with the bias of the futures price. Figure 2

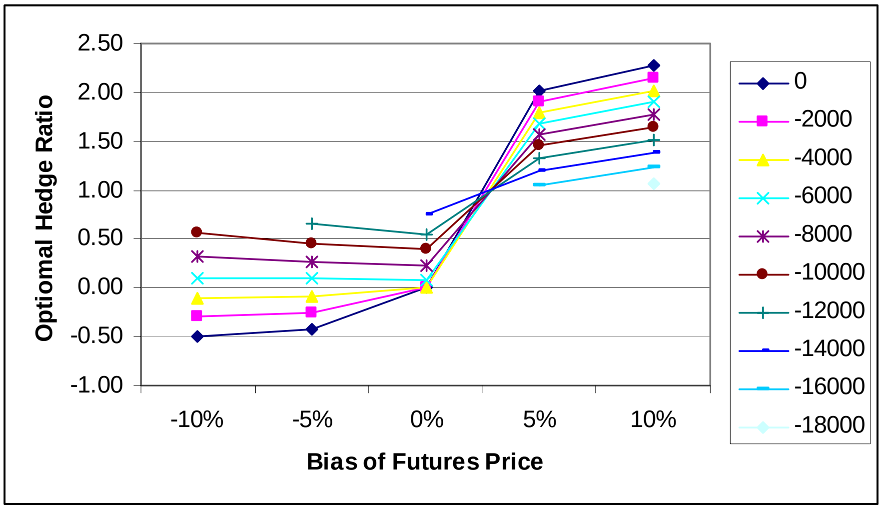

Figure 2. Bias of futures price versus the optimal hedge ratio curves associated with different 90% CVaR upper bounds in the La Niña phase

shows the bias of futures price versus the optimal hedge ratio curves associated with different 90% CVaR upper bounds in the La Niña phase. When the CVaR constraint is not strict (i.e. the upper bound of 90%CVaR equals zero) the optimal hedge ratio curve follows the pattern claimed in Mahul (2003). The hedge ratio increases (decreases) with the positive (negative) bias of futures price in a decreasing rate. However, when the CVaR constraint becomes stricter (i.e., the CVaR upper bound equals -$8,000), the optimal hedge ratio increases not only with the positive bias but also with the negative one. This is because the higher negative bias of futures price implies a heavier cost (loss) is involved in futures hedge. It makes the CVaR constraint become stricter, requiring a higher pure hedge component to satisfy the constraint. The net change of the optimal hedge ratio, including an increment in the pure hedge component and a decrement in the speculative component, depends on the loss distribution, the CVaR upper bound, and the bias of futures price.

Table 6. Optimal Planting Schedule for Different Biases of Futures Price in the Three ENSO Phases

Bias	90%CVaR upper bound	El Niño Date1	El Niño Date2	El Niño Date3	El Niño Date4	Neutral Date1	Neutral Date2	Neutral Date3	Neutral Date4	La Niña Date1	La Niña Date2	La Niña Date3	La Niña Date4
	-20000		x				x						
	-18000	100	0	0	0	0	0	97	3				
-10%	-16000	100	0	0	0	0	0	100	0				
	-12000	100	0	0	0	0	0	100	0			x	
	0 to -8000	100	0	0	0	0	0	100	0	0	0	0	100
	-24000		x				x						
	-20000	100	0	0	0	0	0	39	61				
	-18000	100	0	0	0	0	0	100	0				
-5%	-16000	100	0	0	0	0	0	100	0				
	-14000	100	0	0	0	0	0	100	0			x	
	-12000	100	0	0	0	0	0	44	56	0	0	0	100
	-8000	100	0	0	0	0	0	85	15	0	0	0	100
	-4000	100	0	0	0	0	0	92	8	0	0	0	100
	0	100	0	0	0	0	0	93	7	0	0	0	100
	-24000		x				x						
0%	-20000	100	0	0	0	0	0	100	0				
	-16000	100	0	0	0	0	0	100	0			x	
	0 to -12000	100	0	0	0	0	0	100	0	0	0	0	100
	-28000		x				x						
	-24000	100	0	0	0	0	0	47	53				
	-20000	100	0	0	0	0	0	68	32			x	
5%	-16000	100	0	0	0	0	0	92	8	38	0	0	62
	-12000	100	0	0	0	0	0	100	0	35	0	0	65
	-8000	100	0	0	0	0	0	100	0	33	0	0	67
	-4000	100	0	0	0	0	0	100	0	38	0	0	62
	0	100	0	0	0	0	0	100	0	37	0	0	63
	-32000						x						
	-28000		x			0	0	31	69				
	-24000	100	0	0	0	0	0	54	46				
	-20000	100	0	0	0	0	0	75	25			x	
10%	-18000	100	0	0	0	0	0	87	13	33	17	0	50
	-16000	100	0	0	0	0	0	96	4	45	0	0	55
	-12000	100	0	0	0	0	0	100	0	47	0	0	53
	-8000	100	0	0	0	0	0	100	0	54	0	0	46
	-4000	100	0	0	0	0	0	100	0	59	0	0	41
	0	100	0	0	0	0	0	100	0	63	0	0	37

Note: Negative CVaR upper bounds represent profits. Four planting dates: 'Date1' = April 16, 'Date2' = April 23, 'Date3' = May 1, 'Date4' = May 8. "x" represents infeasible.

Table 6 shows the optimal planting schedules for different biases of futures price in the three ENSO phases. For the El Niño phase, the optimal planting schedule (i.e., planting

100 acres in date 1) was not affected by the biases of futures price and 90%CVaR upper bounds. For the Neutral phase, however, the optimal planting strategy was to plant on date 3 and/or date 4 depending on the 90% CVaR upper bounds. More specifically, date 3 is the optimal planting date for all risk tolerances under unbiased futures market. When futures prices are positive biased, the lower the 90%CVaR upper bounds (i.e., the stricter the CVaR constraint), the more planting acreages moved to date 4 from date 3. This result was based on the fact that there was no insurance coverage involved in the optimal hedging strategy. When the futures prices are negative biased, the optimal planting schedule had the same pattern as positive biased futures markets but was affected by the existence of insurance coverage in the optimal hedging strategy. For example, when the 90%CVaR upper bounds were within the range of -$18,000 and -$14,000 under a -5% biased futures price, the optimal planting acreage on date 4 went down to zero due to a 75%CRC in the optimal hedging strategy. For La Niña, the optimal planting schedule was to plant 100 acres on date 4 when future prices were unbiased or negative biased. When the future price was negative biased, the stricter the CVaR constraint, the more planting acreage is shifted from date 4 to date 1. For deep positive biased futures price together with a strict CVaR constraint (i.e., 10% biased futures price and -$18,000 90%CVaR upper bound), the optimal planting schedule included date 1, 2, and 4.

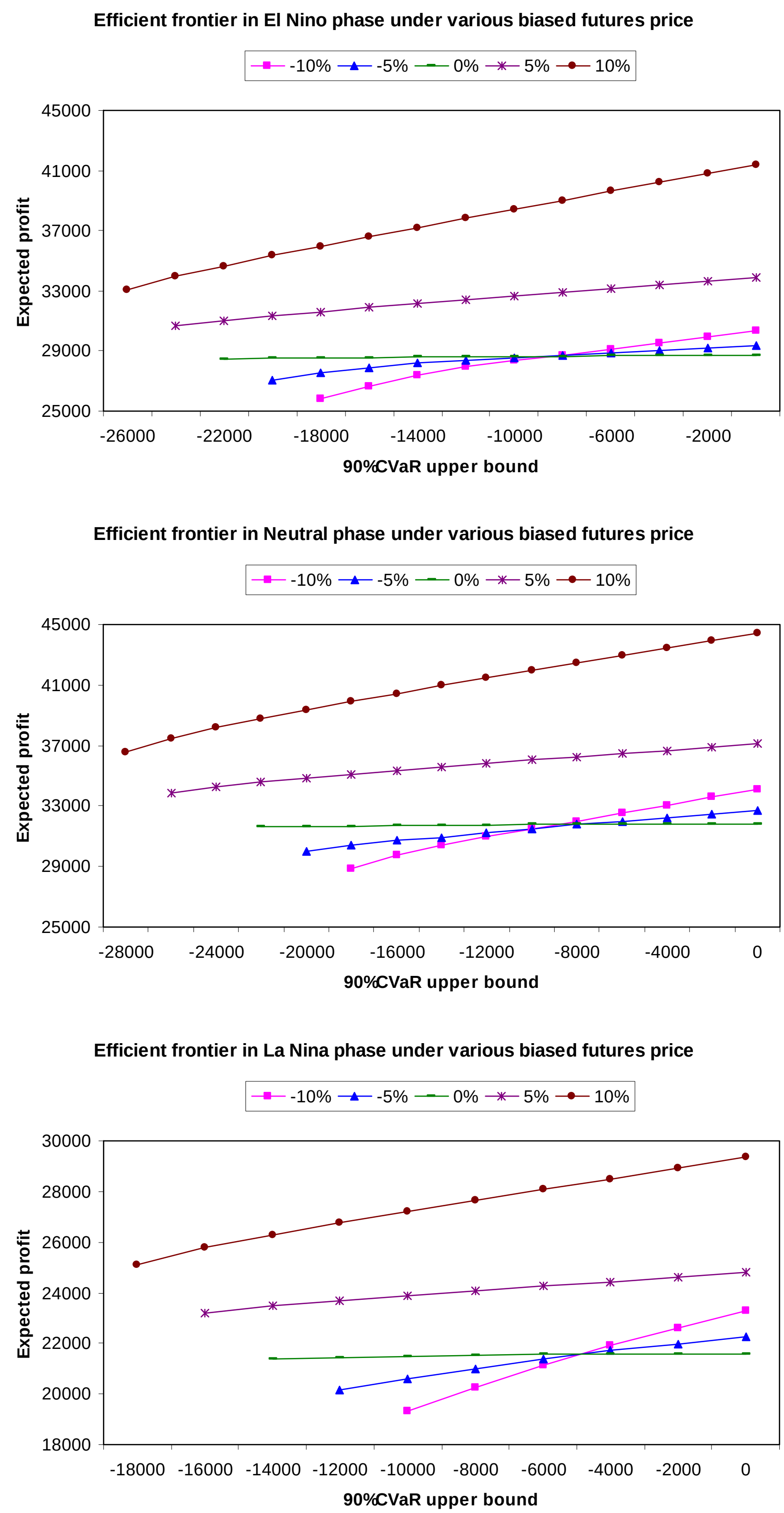

Figure 3. The efficient frontiers under various biased futures price in the three ENSO phases

Figure 3 shows the mean-CVaR efficient frontiers associated with various biased futures prices for different ENSO phases. With the efficient frontiers, the farmer makes the optimal decision based on the trade off between expected profit and downside risk while staying within his/her downside risk tolerance. The three graphs show that the Neutral phase has the highest expected profit and lowest feasible CVaR upper bound. In contrast, the La Niña phase has the lowest expected profit and highest feasible CVaR upper bound. The pattern of the efficient frontiers in the three graphs is the same. The higher the positive bias of the futures price, the higher the expected profit. However, the higher negative bias of futures price provides a higher expected profit under a looser CVaR constraint and a lower expected profit under a stricter CVaR constraint.

Conclusion

This article proposed a mean-CVaR model for investigating the optimal crop planting schedule and hedging strategy when crop insurances and/or futures contracts are available for hedging the yield and price risk. Due to the linear property of CVaR, the optimal planting and hedging problem could be formulated as a mixed 0-1 linear programming problem that could be efficiently solved by many commercial solvers such as CPLEX. The mean-CVaR model is powerful in the sense that it inherits the advantage of the return versus risk framework (Markowitz, 1952) and further utilizes CVaR as a (downside) risk measure that can cope with general loss distributions. Compared to using utility functions for modeling risk aversion, the mean-CVaR model provides a more intuitive way to define risk. In addition, a problem without nonlinear side constraints could be formulated linearly under the mean-CVaR framework, which could be solved more efficiently compared to the nonlinear formulation from the utility function framework.

A case study was conducted using the data of a representative cotton producer in Jackson County, Florida to examine the optimal crop planting schedule and risk hedging strategy under the three ENSO phases. The available hedging instruments for cotton include futures contracts and three types of crop insurance policies: APH, CRC, and CAT. When crop insurances are the only available hedging instruments, the 65%CRC or 70%CRC would be the optimal insurance coverage as the CVaR constraint becomes strict. Furthermore, the optimal hedging strategy was examined when crop insurance policies and futures contracts are available. When the futures price is unbiased or positive biased, no crop insurance policy is desirable and the optimal hedging strategy only includes futures contracts. However, when the futures price is negative biased, the optimal hedging strategy depends on the ENSO phases. In the El Niño phase, the optimal hedging strategy consists of the 65%CRC (or 70%CRC for some CVaR upper bounds) and futures contracts for all CVaR upper bound values. In the Neutral phase, when the futures price is very negative biased (-10%), the optimal hedging strategy consists of the 70%CRC and futures contracts for all CVaR upper bound values. Under a -5% biased futures price, the optimal hedging strategy includes the 70%CRC and futures contracts when the CVaR upper bound is within the range of -$18,000 and -$14,000. Otherwise, the optimal hedging strategy includes only futures contract. In the La Niña phase, the optimal hedging strategy contains only futures contract for all CVaR upper bound values and all biases of futures prices between -10% and 10%. The optimal futures hedge ratio increases with the CVaR upper bound when the insurance strategy is unchanged. For a fixed CVaR upper bound, the optimal hedge ratio increases when the positive bias of futures price increases. However, when the futures price is negative biased, the optimal hedge ratio depends on the value of CVaR upper bound.

The case study provides some insight into how planting schedule, insurance and futures hedging may manipulate the downside risk of a loss distribution. The small sample size for the El Niño and La Niña phases may limit the case study results. In addition, the cost of futures contract was assumed to be the commission plus an average interest foregone for margin deposit. The risk of daily settlement, which may require a large amount of cash for margin account, was not considered. This may reduce the value of futures hedging for risk-averse farmers.

References

Artzner, P., F. Delbaen, J.-M. Eber, and D. Heath. 1999. "Coherent measures of risk". *Mathematical Finance*, 9, 203–228.

Cabrera, V.E., C. Fraisse, D. Letson, G. Podestá, and J. Novak. 2006. Impact of climate information on reducing farm risk by optimizing crop insurance strategy. Transactions of the American Society of Agricultural and Biological Engineers (ASABE) 49: 1223-1233.

Cabrera, V.E., D. Letson, and G. Podestá. 2007. The Value of the Climate Information when Farm Programs Matter. Agricultural Systems 93: 25-42.

Cane, M.A., G. Eshel, and R. Buckland. 1994. "Forecasting Zimbabwean maize yield using eastern equatorial Pacific sea surface temperature." *Nature* 370, 204-205.

Chambers, R.G. and J. Quiggin. 2002. "Optimal producer behavior in the presence of area-yield crop insurance." *American Journal of Agricultural Economics*. 84, 320-334.

Coble, K. H., J.C. Miller, and M. Zuniga. 2004. "The joint effect of government crop insurance and loan programmes on the demand for futures hedging." *European Review of Agricultural Economics*. 31, 309-330

Hansen, J.W., A.W. Hodges, and J.W. Jones. 1998. "ENSO influences on agriculture in the Southeastern US." *Journal of Climate* 11, 404-411.

Hansen, J.W. 2002. "Realizing the potential benefits of climate prediction to agriculture: issues, approaches, challenges." *Agricultural Systems* 74, 309-330.

Jones, J.W., J.W. Hansen, F.S. Royce, and C.D. Messina. 2000. "Potential benefits of climate forecast to agriculture." *Agriculture, ecosystems & environment.* 82, 169-184.

Jones, J.W., G. Hoogenboom, C.H. Porter, K.J. Boote, W.D. Batchelor, L.A. Hunt, P.W. Wilkens, U. Singh, A.J. Gijsman, J.T. Ritchie. 2003. "The DSSAT cropping system model." *European Journal of Agronomy.* 18, 235-65.

Jorion, P. 2000. *Value at Risk: The New Benchmark for Managing Financial Risk*, 2nd. ed. New York: McGraw-Hill Book Co.

Kruksal, W.H. 1958. "Ordinal measures of association." *Journal of the American Statistical Association*. 53, 814-861.

Mahul O. 2003. "Hedging Price Risk in the Presence of Crop Yield and Revenue Insurance." *European Review of Agricultural Economics*, 30, 217-239.

Markowitz, H.M. 1952. "Portfolio Selection." *Journal of Finance*, 7, 77–91.

McKinnon, R. I. 1967. "Futures Market, buffer stocks, income stability for primary producers." *Journal of Political Economy.* 75, 844-861.

Messina, C.R., J.W. Jones, C.W. Fraisse. 2005. "Development of cotton CROPGRO crop model." *Southeast Climate Consortium Staff Paper Series 05-05*. Gainesville, FL.

Nemhauser, G.L. and L.A. Wolsey. 1999. *Integer and Combinatorial Optimization*, New York: Wiley Book Co.

Poitras, G. 1993. "Hedging and Crop Insurance." *The Journal of Futures Markets.* 13, 373-389.

Rockafellar R.T. and S. Uryasev. 2002. "Conditional value-at-risk for general loss distribution." *Journal of Banking and Finance.* 26, 1443-1471.

——. 2000. "Optimization of Conditional Value-At-Risk." *The Journal of Risk*, 2, 21-41.

Sklar A. 1959. "Fonctions de répartition à *n* dimensions et leures marges." *Publications de l'Institut de Statistique de L'Université de Paris.* 8, 229-231.

Szegö G. 2002. "Measures of Risk." *Journal of Banking and Finance*, 26, 1253–1272.