α-quantile option in a jump-diffusion economy

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Lookback options represent a typology of path-dependent contracts with payoff determined by the maximum or the minimum price of the underlying asset within the life of the option. In particular, a fixed-strike lookback call pays the highest value achieved by the stock price during the contract lifetime; hence it can be considered by an investor as an insurance against large downward movements of the stock near maturity, and might be thought of as a way to deal with the market exit problem. Analogously, the lookback put provides a protection from substantial rises in the market near expiration. However, these features make lookback options too expensive and hence not attractive to ordinary investors. To overcome this limitation new types of exotic contracts came to existence in the attempt of reducing the option price whilst preserving its potential payoff. Examples of new lookback-type contracts are the partial lookback options introduced by Heynen and Kat (1994), which are characterized by a monitoring period for the extreme value of the underlying asset price to be a subset of the option's lifetime; and the fractional lookback, for which only a percentage of the extreme values is in effect in the payoff function of the options.

Another possible alternative contract could be identified in the α-quantile option introduced by Miura (1992). An α-quantile call option with strike K and underlying asset S has a payoff function at maturity T defined as $\left(S_0 e^{Q(\alpha,T)} - K\right)^+$, where S_0 is the value of the underlying asset at the beginning of the contract and $Q\left(\alpha, T\right)$ is the α-quantile of the process X, driving the underlying asset price S. In particular Q is defined to be the smallest level below which the process spends at least a fraction $\alpha \in (0,1)$ of some period $\left[0, T\right]$, that is

$$Q\left(\alpha, T\right) = \inf\left\{x : \int_0^T 1_{(X_t \le x)} dt > \alpha T\right\}.$$

It follows from the definition that

$$\lim_{\alpha \to 1} Q\left(\alpha, T\right) = \sup_{0 \le t \le T} X_t$$

and

$$\lim_{\alpha \to 0} Q\left(\alpha, T\right) = \inf_{0 \le t \le T} X_t.$$

Using this property, Ballotta and Kyprianou (2000) have shown that the α-quantile option is comparatively cheaper than the fixed strike lookback written on the same underlying and with monitoring period equal to the contract lifetime. Precisely this feature suggests an interesting potential use of this path-dependent contract, introduced mainly as a "mathematical exercise" and not yet traded in the market. In fact, it might be seen as a valid tool to generate at maturity returns similar to the lookback option ones but for a less expensive initial investment. Since the convergence of the α-quantile option price to the price of the equivalent lookback, the investor has also the possibility to increase and control the leverage effect of his portfolio in a quite flexible way through a suitable setting of the parameter α.

Close pricing formulas for the α-quantile option have been derived in the Brownian motion setting by both Akahori (1995) and Dassios (1995). However, these valuation formulas present serious computational difficulties because they are still expressed in integral form. Ballotta and Kyprianou (2000) have implemented a numerical valuation procedure that removes some of these problems by taking advantage of the Dassios-Port-Wendel identity, which expresses the distribution of the α-quantile of a Brownian motion as the convolution of the supremum and the infimum of the process itself.

The aim of this communication is to analyze the behaviour of the α-quantile option in a more realistic setup for the market model. Precisely, we will consider a general Lévy motion as relevant process for the price of the underlying security and we will derive the price of the α-quantile option in such a framework. We will also introduce a Monte Carlo simulation procedure for this price and, since a jump-diffusion economy identifies in general an incomplete market, such a procedure will be extended to price the option under different risk-neutral martingale measures. Particular attention will be given to the mispricing generated by the misspecification of a jump-diffusion process for the underlying asset as a pure diffusion process.