Games, Incompetence, Training, and Related Parametric Analysis

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Abstract

In classical strategic game theory the pay-offs are fully determined directly by players' choices of strategies. However, a player may not be capable of executing a chosen strategy due to lack of skill/capability otherwise known as "incompetence". A common approach to dealing with incompetence is to devote resources to "training/learning" that may be in the form of investment in new technologies or capabilities. In practice, such efforts will rarely eliminate incompetence completely but can reduce it to more acceptable levels, especially if other players do not invest in similar incompetence reduction strategies. Of course, in general, all participants may engage in these training activities and the problem of how to model and assess the benefits (if any) of these training efforts leads to many fascinating questions in game theory and optimization as well as to some, mathematically very challenging, parametric analysis problems. Indeed, it is natural to conjecture that - depending on the structure of solutions to these analyses - incompetence may easily induce "tacit", or even formal, cooperation in the behavior of otherwise non-cooperative players. Arguably, anti-ballistic missile treaties are manifestations of this induced cooperation.

In this presentation we outline some approaches to the modeling, analysis, and solution of instances of this difficult class of problems. In particular, a method for examining incompetence in certain games is introduced, examined, and illustrated. Along with the derivation of general characteristics, a number of interesting special behaviors are identified. The latter are shown to be the result of special forms of the game and/or the incompetence matrices. A simplified application to capability investment decisions in the military is discussed.

In addition, we consider a class of parametric optimization problems that arise naturally in this context. We demonstrate that in certain situations, even small changes in a parameter can lead to dramatic changes in optimal solutions. We outline a recent, unified, theory for determining when this phenomenon arises. Surprisingly, perhaps, this theory relies on some powerful results and techniques - such as those of Gröbner bases and complex algebraic varieties - developed by pure mathematicians but rarely used in the kind of practical applications that stimulated these investigations.