# Computational Methods for a Class of Discrete Valued Optimal Control Problems Arising in Defense Applications

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**Abstract**

We consider a general class of optimal control problems where the control functions are assumed piecewise constant and only take on values from a finite discrete set. The aim in such a problem is to find a sequence of discrete control values and a corresponding set of exact switching times (i.e., times where control should switch between the discrete values) such that a given functional representing cost or risk is minimized. Such problems arise in a range of applications. One application is the ''Transit Path Problem'', where an object such as a robot or vehicle (air, naval, space or land) needs to traverse a specified region (discrete or continuous) between two points in a prescribed time so as to avoid detection. The objective is to find a path for the object which satisfies the time constraints and which minimizes the total risk of detection. The risk function is not simple and depends on a range of factors such as the environment, the types of sensors, the speed, direction, and position of the vehicle.

The main difficulty with these problems is that the range of some of the controls is discrete and hence not convex. Since the gradients with respect to the switching time parameters are discontinuous, ordinary gradient based solution methods perform poorly. An additional difficulty is to determine exactly how many switching times are involved in an optimal solution. We address the first difficulty by using the Control Parameterization Enhancing Transform (CPET) and the second difficulty by solving a sequence of problems which are transformed via CPET.

With respect to the transit path problem our strategy involves a two stage approach. The first stage involves a discretization of the problem and the solution of a constrained path problem in a network. The second stage involves the use of an optimal control model and a solution procedure that utilizes the solution obtained from the first stage.

In this talk we present a range of models and solution methods as well as discuss in detail a number of applications arising in the defense area.