Technologies Leading to Unified Multi-Agent Collection and Coordination

AFOSR Cooperative Control Theme
Contract F49620-01-C-0031
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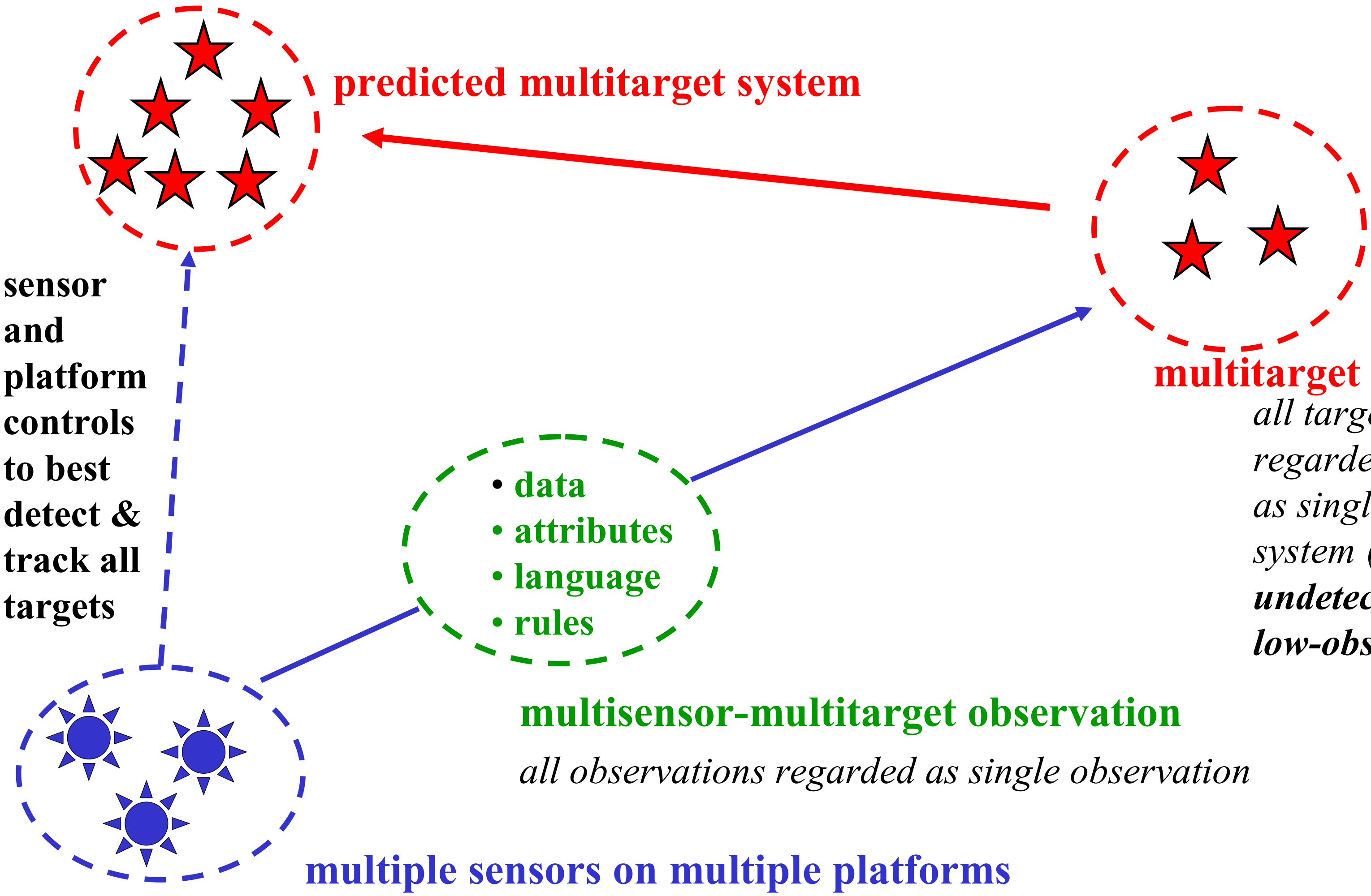
Subcontractor: *Scientific Systems Co., Inc., Woburn MA*

November 14, 2001
Workshop for Cooperative Control and Optimization
University of Florida, Gainesville FL

Objective: Integrated Collection/Coordination



predicted multitarget system

sensor and platform controls to best detect & track all targets

- **data**
- **attributes**
- **language**
- **rules**

multitarget system

all targets regarded as single system (possibly ***undetected low-observable****)*

multisensor-multitarget observation

all observations regarded as single observation

multiple sensors on multiple platforms

all sensors on all platforms regarded as single system

Topics

- **Summary of overall approach**
- **Cooperative collection approach**
  - *recursive Bayes nonlinear filtering*
  - *multi-object statistics*
  - *multisensor-multitarget filtering*
  - *multisensor-multitarget sensor management*

Multi-Agent Coordination: Overall Approach

Novel integration of three major approaches:

- leader following

- behaviorial

- virtual structure

Decentralized formation control architecture

- each platform has own coordination variable, controller

- determine correct synchronization & convergence

Autonomous intelligent adaptive controller algorithms

Path planning

determine feasible paths

Trajectory generation

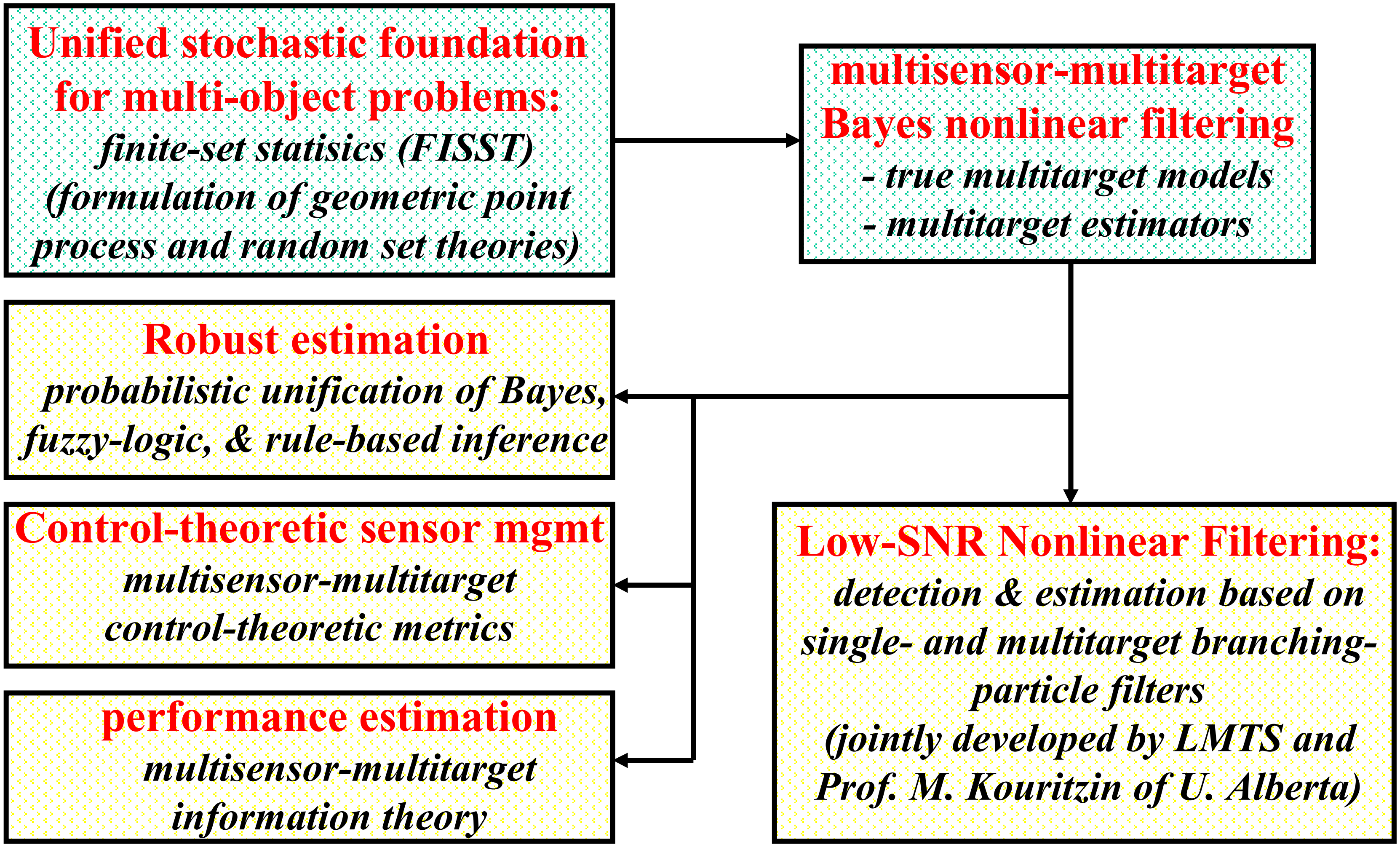
generate temporal paths

Formation hold

initialize & maintain formation

Presentation following this one: "Mixed Integer/ LMI Programs for Low-Level Path Planning," R.Prasanth, J. Boskovic, and R. Mehra

Multi-Agent Collection: Overall Approach



Unified stochastic foundation for multi-object problems:
finite-set statistics (FISST)
(formulation of geometric point process and random set theories)

multisensor-multitarget Bayes nonlinear filtering
- *true multitarget models*
- *multitarget estimators*

Robust estimation
probabilistic unification of Bayes, fuzzy-logic, & rule-based inference

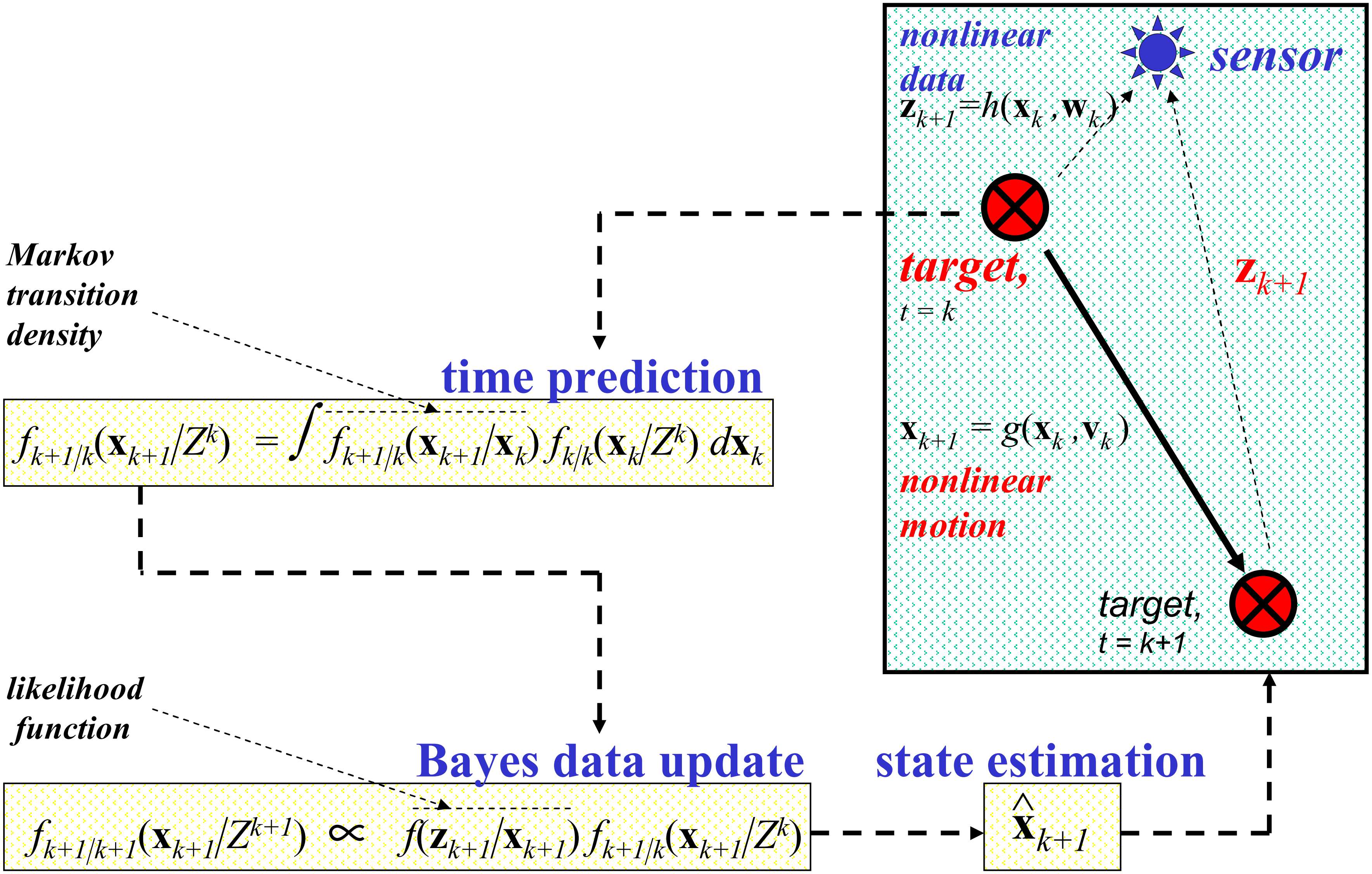
Control-theoretic sensor mgmt
multisensor-multitarget control-theoretic metrics

performance estimation
multisensor-multitarget information theory

Low-SNR Nonlinear Filtering:
detection & estimation based on single- and multitarget branching-particle filters
(jointly developed by LMTS and Prof. M. Kouritzin of U. Alberta)

Bayes Recursive
Nonlinear Filtering (NLF)

Bayes Nonlinear Filtering (NLF)



Markov transition density

time prediction

$$f_{k+1/k}(\mathbf{x}_{k+1}/Z^k) = \int f_{k+1/k}(\mathbf{x}_{k+1}/\mathbf{x}_k)\, f_{k/k}(\mathbf{x}_k/Z^k)\, d\mathbf{x}_k$$

likelihood function

Bayes data update

$$f_{k+1/k+1}(\mathbf{x}_{k+1}/Z^{k+1}) \propto f(\mathbf{z}_{k+1}/\mathbf{x}_{k+1})\, f_{k+1/k}(\mathbf{x}_{k+1}/Z^k)$$

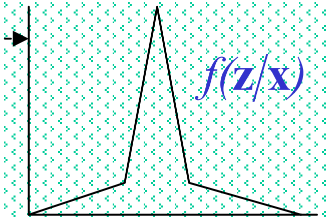
state estimation

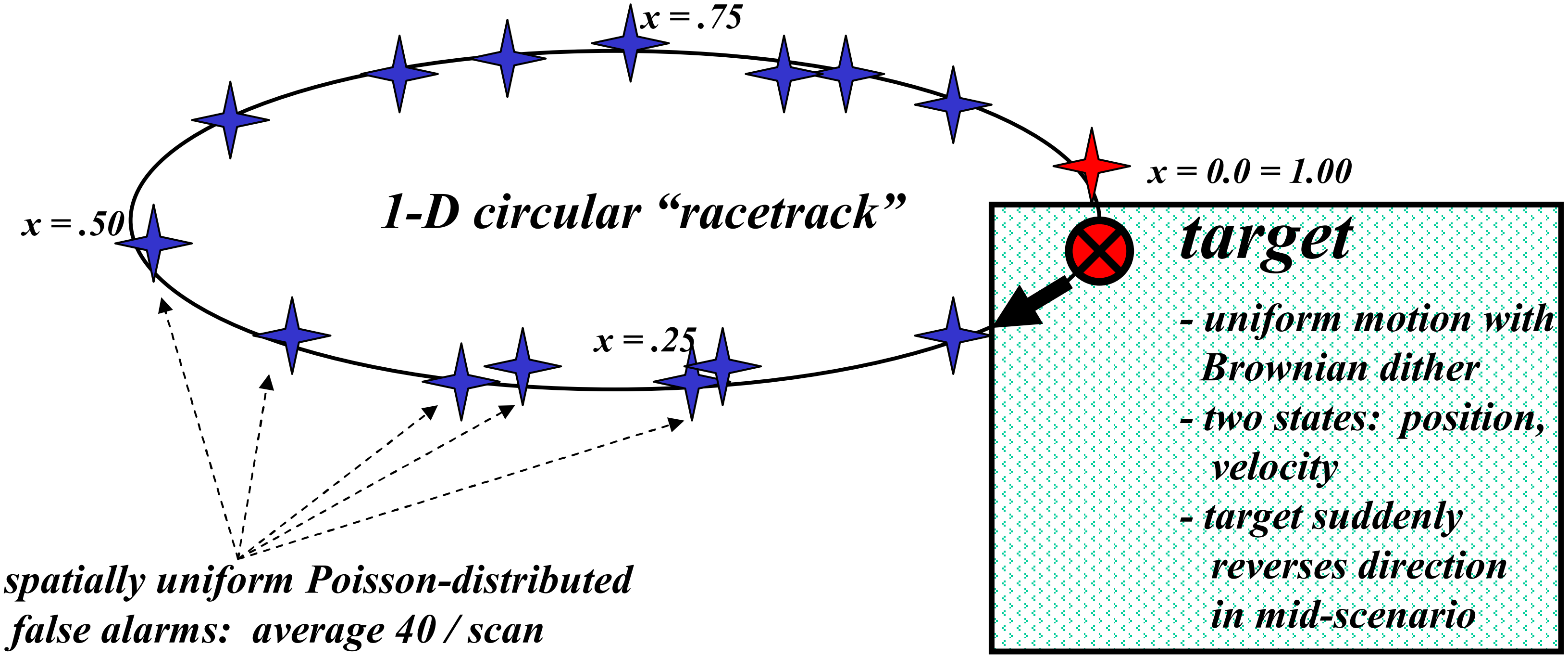
$$\hat{\mathbf{x}}_{k+1}$$

Example: Bayes Filtering in Low SNR

sensor

- *non-Gaussian "witch's hat function" sensor noise*
- *non-unity probability of detection: $p_D = .93$*



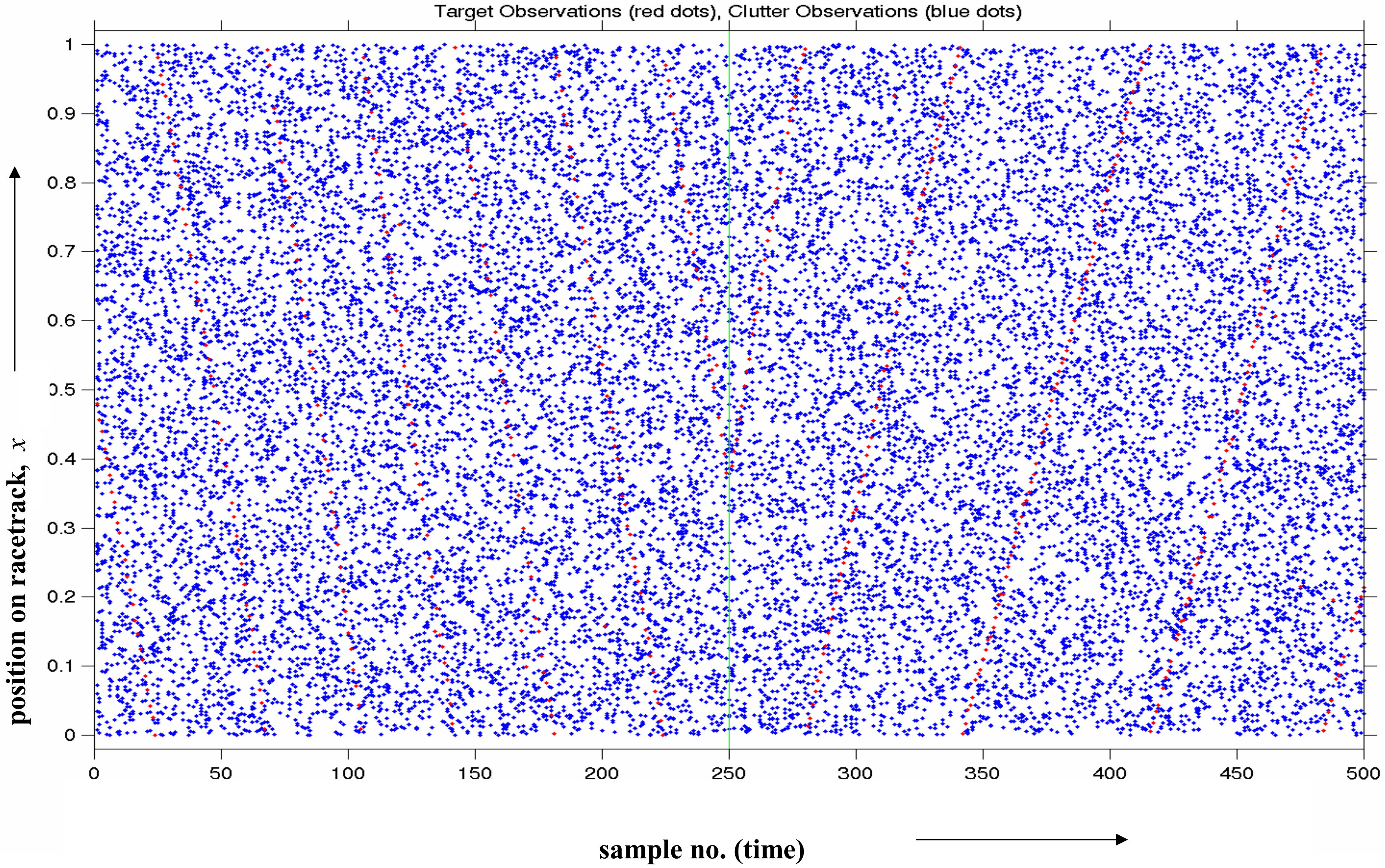


target

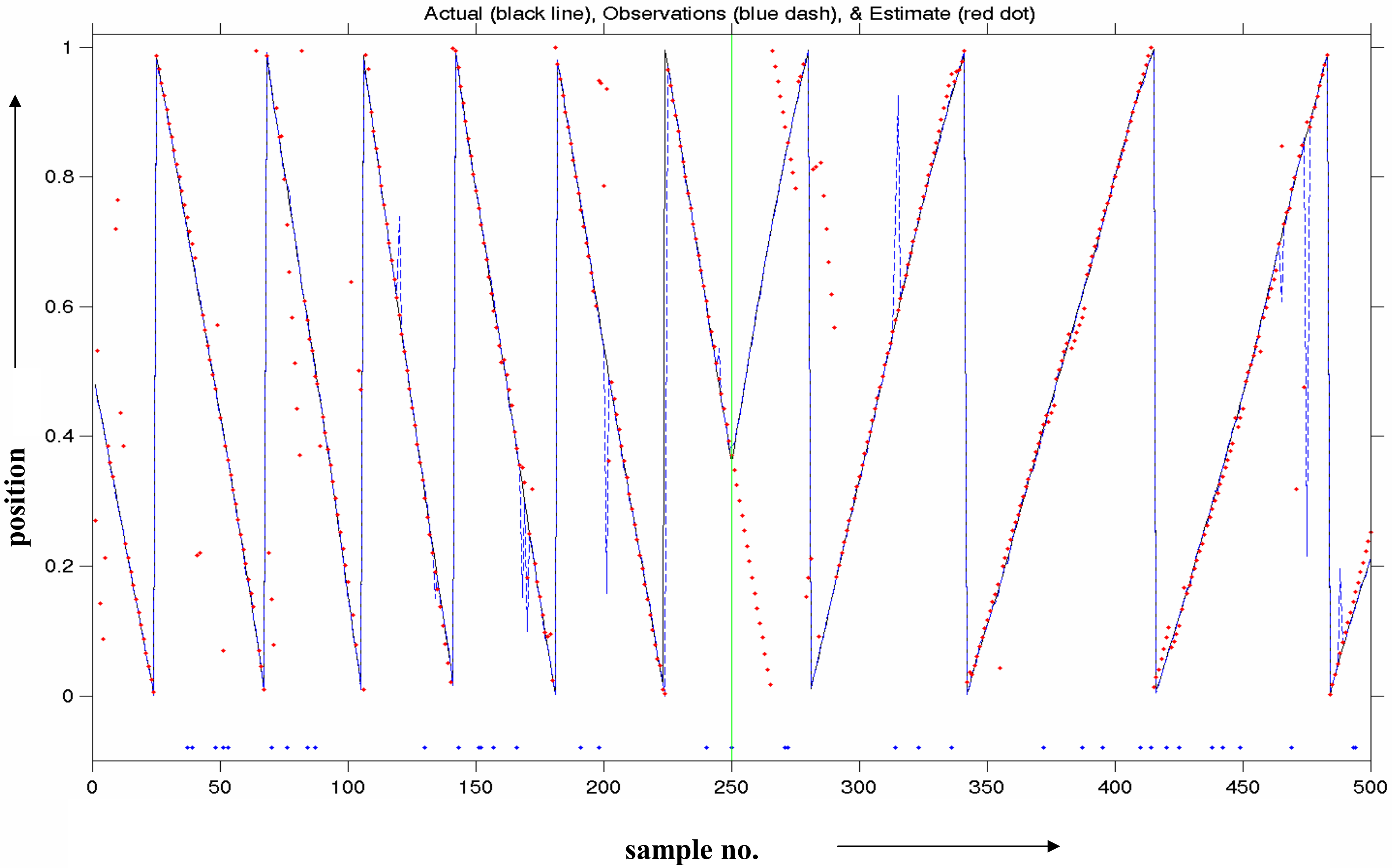
- *uniform motion with Brownian dither*
- *two states: position, velocity*
- *target suddenly reverses direction in mid-scenario*

- *spatially uniform Poisson-distributed false alarms: average 40 / scan*

Input to Bayes Filter

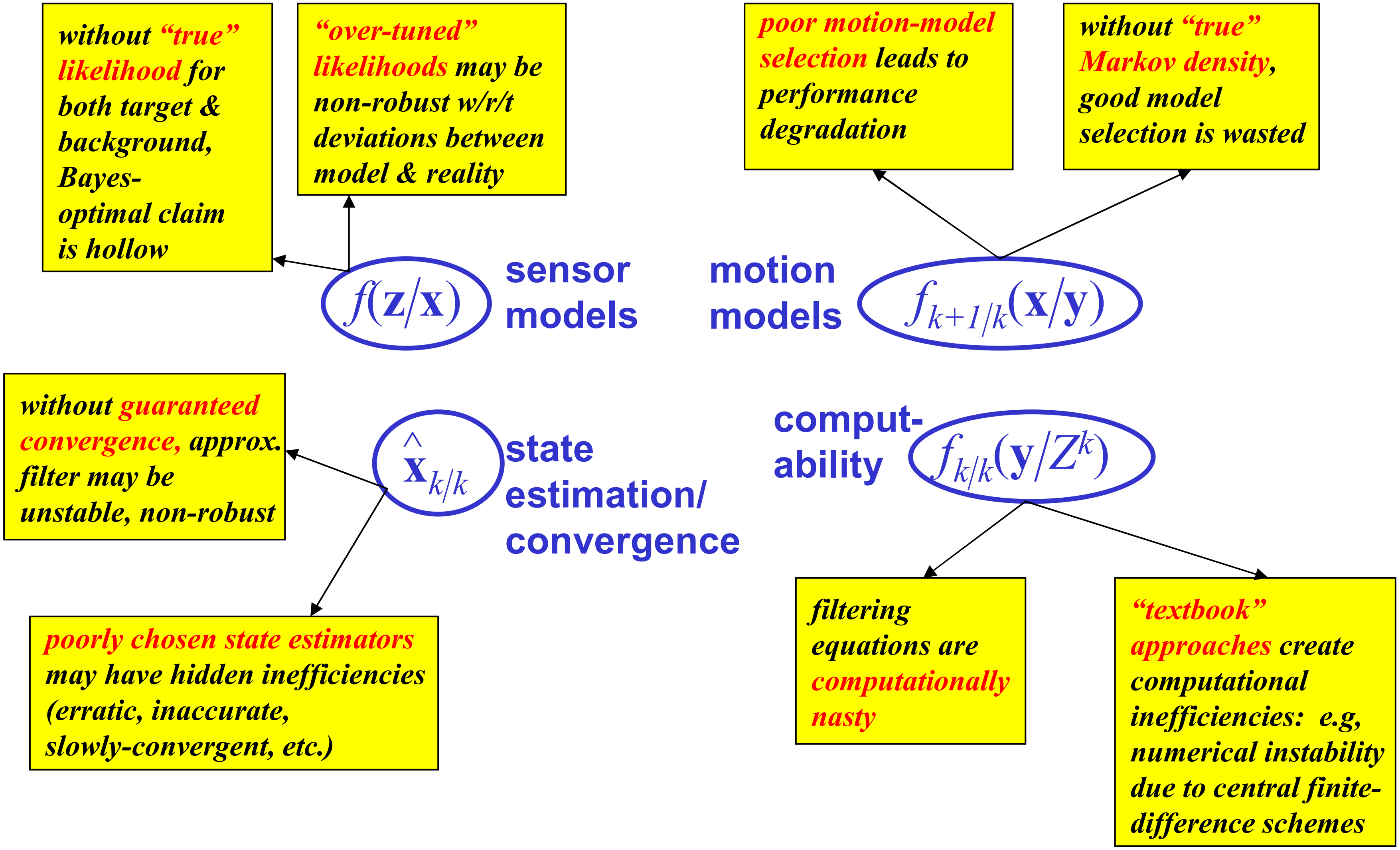


Output of Bayes Filter



Basic Issues in Single-Object Bayes NLF

R. Mahler, “Issues in Non-Linear Filtering (NLF) and NLF Performance Evaluation,”
AFOSR/AFRL Workshop on Nonlinear Filtering Methods for Tracking, Dayton OH, February 22, 2001



sensor models $f(\mathbf{z}/\mathbf{x})$

without “true” likelihood for both target & background, Bayes-optimal claim is hollow

“over-tuned” likelihoods may be non-robust w/r/t deviations between model & reality

motion models $f_{k+1/k}(\mathbf{x}/\mathbf{y})$

poor motion-model selection leads to performance degradation

without “true” Markov density, good model selection is wasted

state estimation/ convergence $\hat{\mathbf{x}}_{k/k}$

without guaranteed convergence, approx. filter may be unstable, non-robust

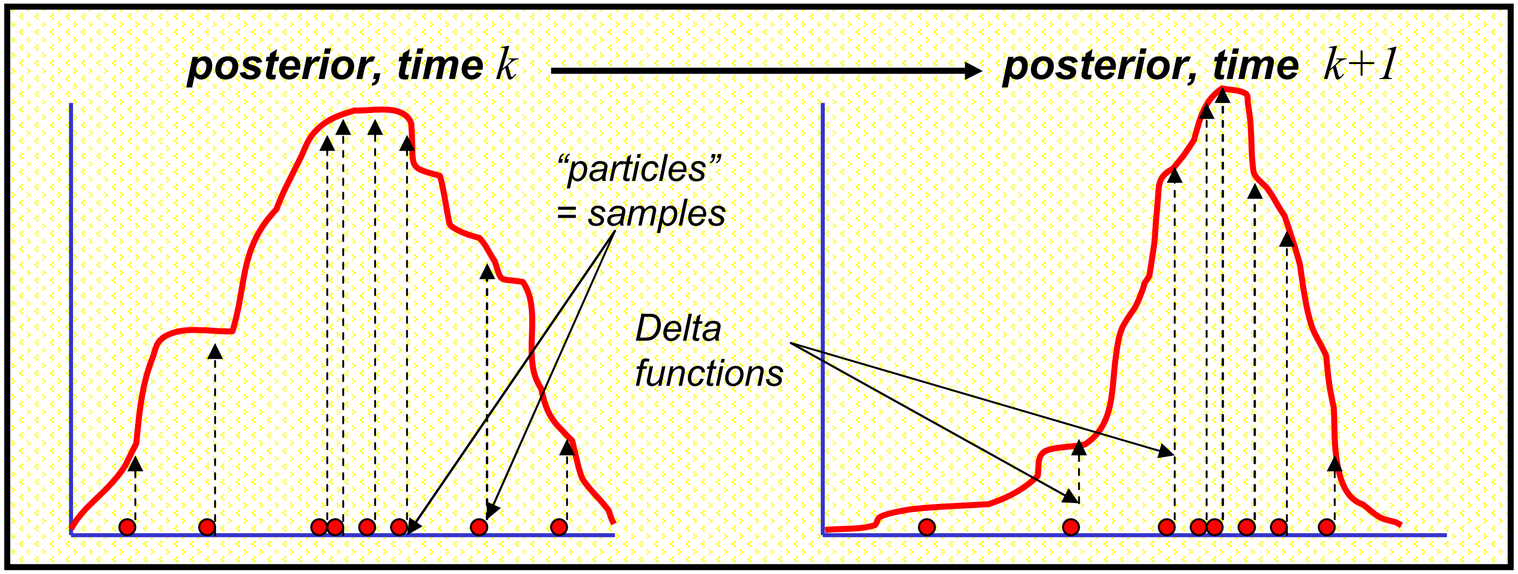
poorly chosen state estimators may have hidden inefficiencies (erratic, inaccurate, slowly-convergent, etc.)

comput-ability $f_{k/k}(\mathbf{y}/Z^k)$

filtering equations are computationally nasty

“textbook” approaches create computational inefficiencies: e.g, numerical instability due to central finite-difference schemes

Particle-System Filters



Non-restrictive w/r/t measurement models

Very general continuous-state Markov models

more general motion models than representable by Fokker-Planck Equation (FPE)

e.g. heavy-tail models, non-smooth models

Very strong, general guaranteed-convergence properties

for every observation sequence, particle distribution converges a.s. to posterior

Computational order: $O(p^d)$ ***(low-SNR detection),*** $O(p)$ ***(low-SNR tracking)***

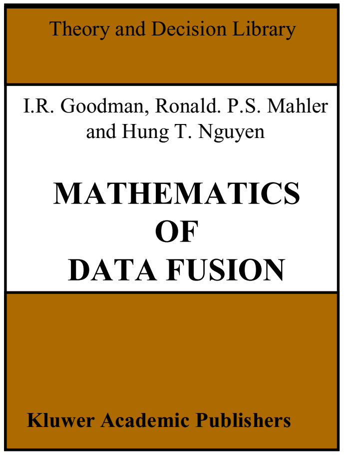
p = no. particles, d = dimensionality, $N = p^d$ = *no. of unknowns*

LMTS is co-developing these filters with U. Alberta (Prof. M. Kouritzin)

Multi-Object Statistics

Finite-Set Statistics (FISST): Background

book



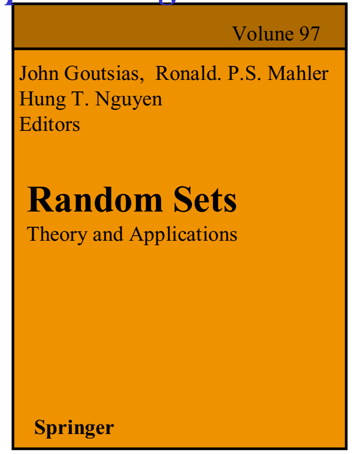
Theory and Decision Library

I.R. Goodman, Ronald. P.S. Mahler
and Hung T. Nguyen

**MATHEMATICS
OF
DATA FUSION**

Kluwer Academic Publishers

hardcover proceedings



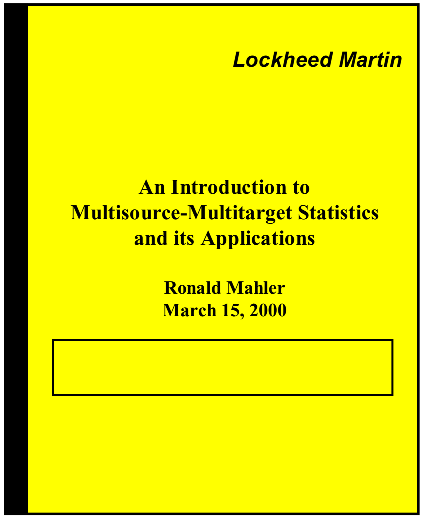
Volume 97

John Goutsias, Ronald. P.S. Mahler
Hung T. Nguyen
Editors

Random Sets
Theory and Applications

Springer

monograph

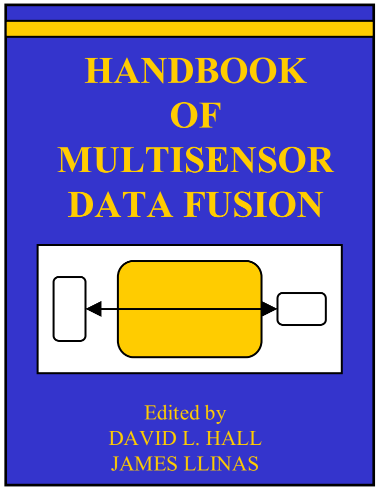


Lockheed Martin

**An Introduction to
Multisource-Multitarget Statistics
and its Applications**

**Ronald Mahler
March 15, 2000**

book chapter



**HANDBOOK
OF
MULTISENSOR
DATA FUSION**

Edited by
DAVID L. HALL
JAMES LLINAS

Scientific Workshops

1995 ONR/ARO/LM Workshop

Invited Session: SPIE AeroSense'99

DoD Advisory

USARO Electr. Div. Technology Planning

USAF Rome Labs Technology Planning

DARPA DDB Program Evaluation

BMDO Project Hercules

Invited Presentations

BMDO/POET

AFIT

NRaD

Harvard

Johns Hopkins

U. Massachusetts

New Mexico State

IDC2002 (Austr.)

ATR Working Group

USAF Correlation Symp.

SPIE AeroSense Conf.

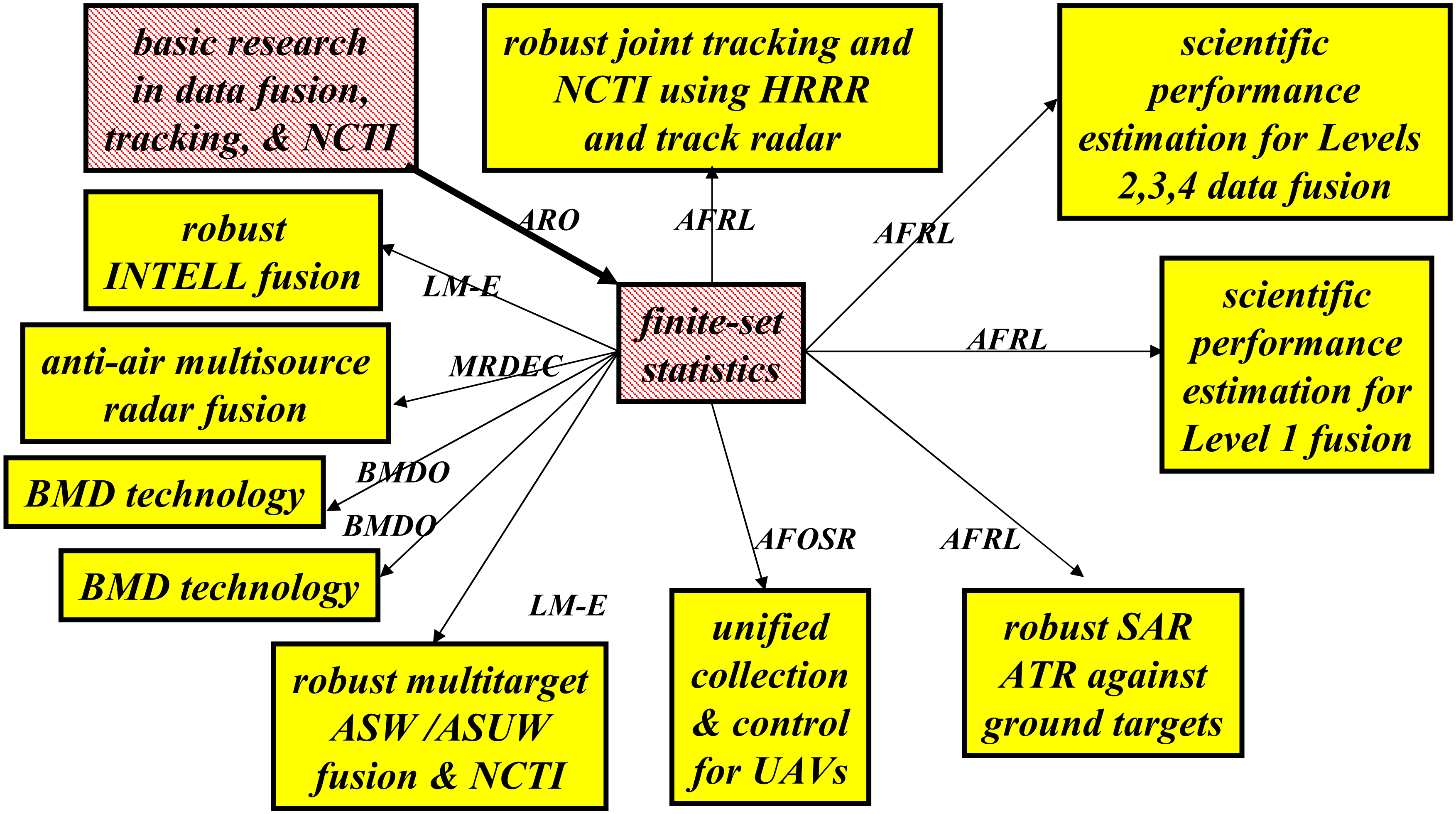
Nat'l Symp. on Data Fusion

IEEE Conf. Dec. & Contr.

Optical Discr. Alg's Conf.

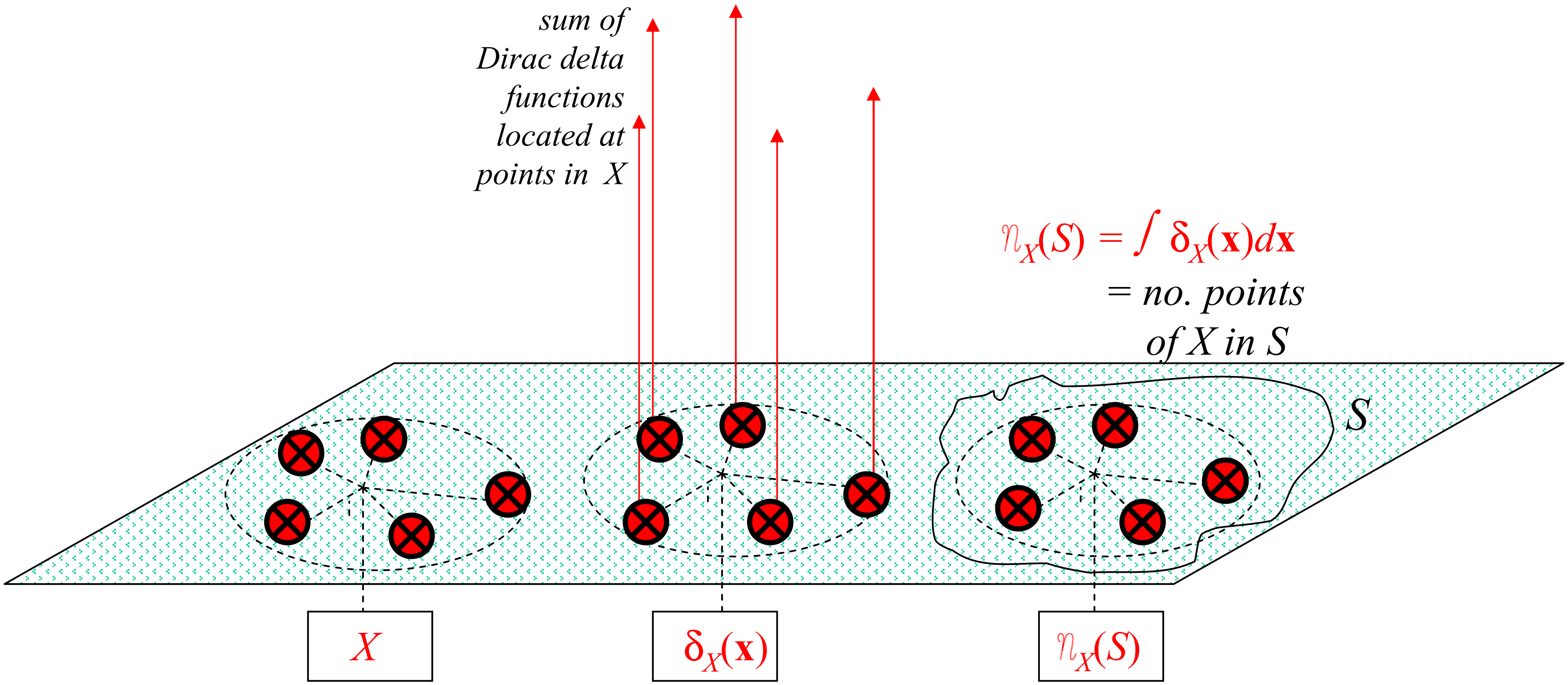
ONR Workshop on Tracking

Finite-Set Statistics: Applied R&D



I. Statistical Foundation of Multi-Object Systems

random finite sets (simple point processes)



sum of Dirac delta functions located at points in X

$\eta_X(S) = \int \delta_X(\mathbf{x})d\mathbf{x}$

= *no. points of* X *in* S

X

random track-set

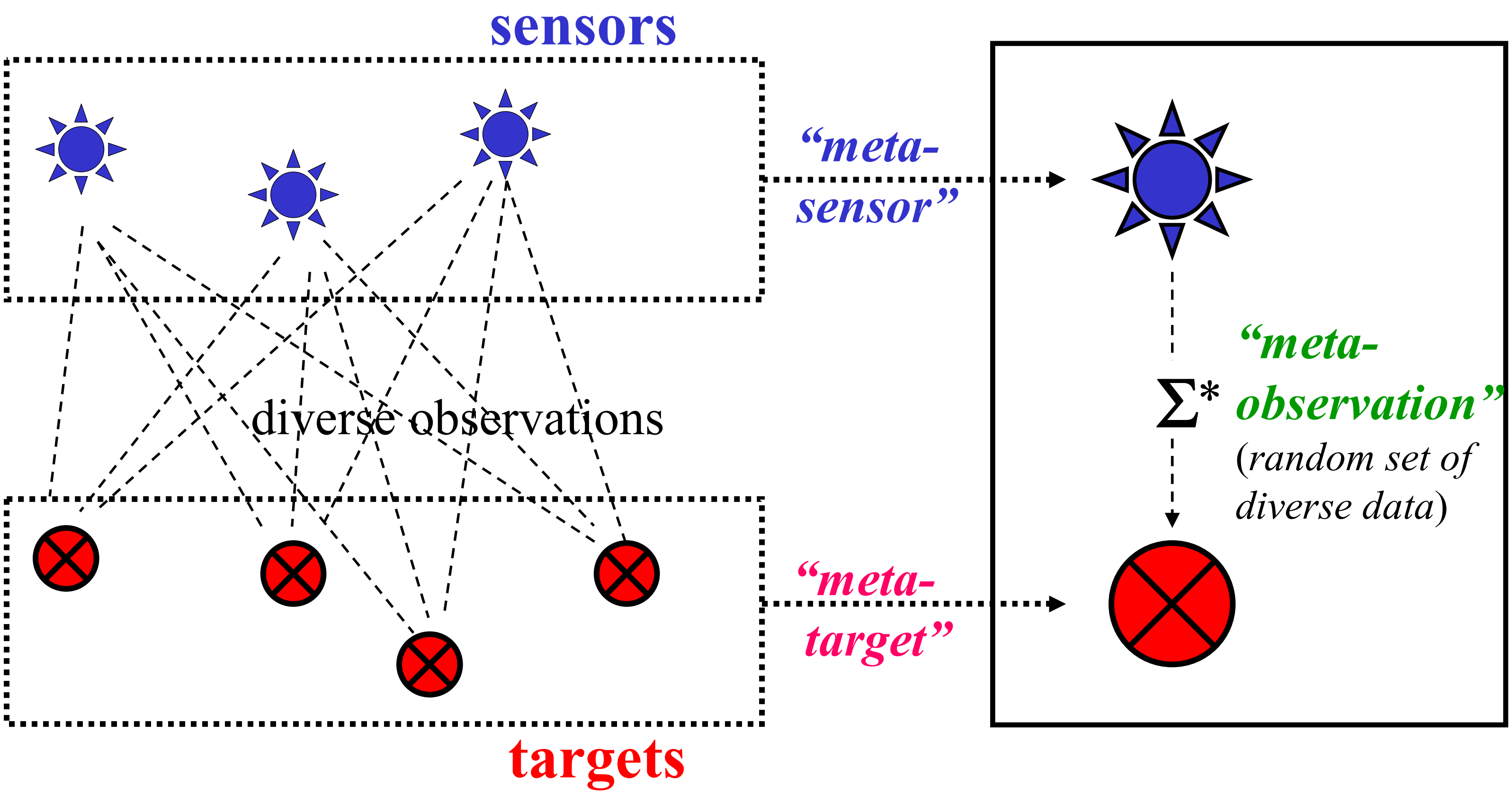
$\delta_X(\mathbf{x})$

random empirical density

$\eta_X(S)$

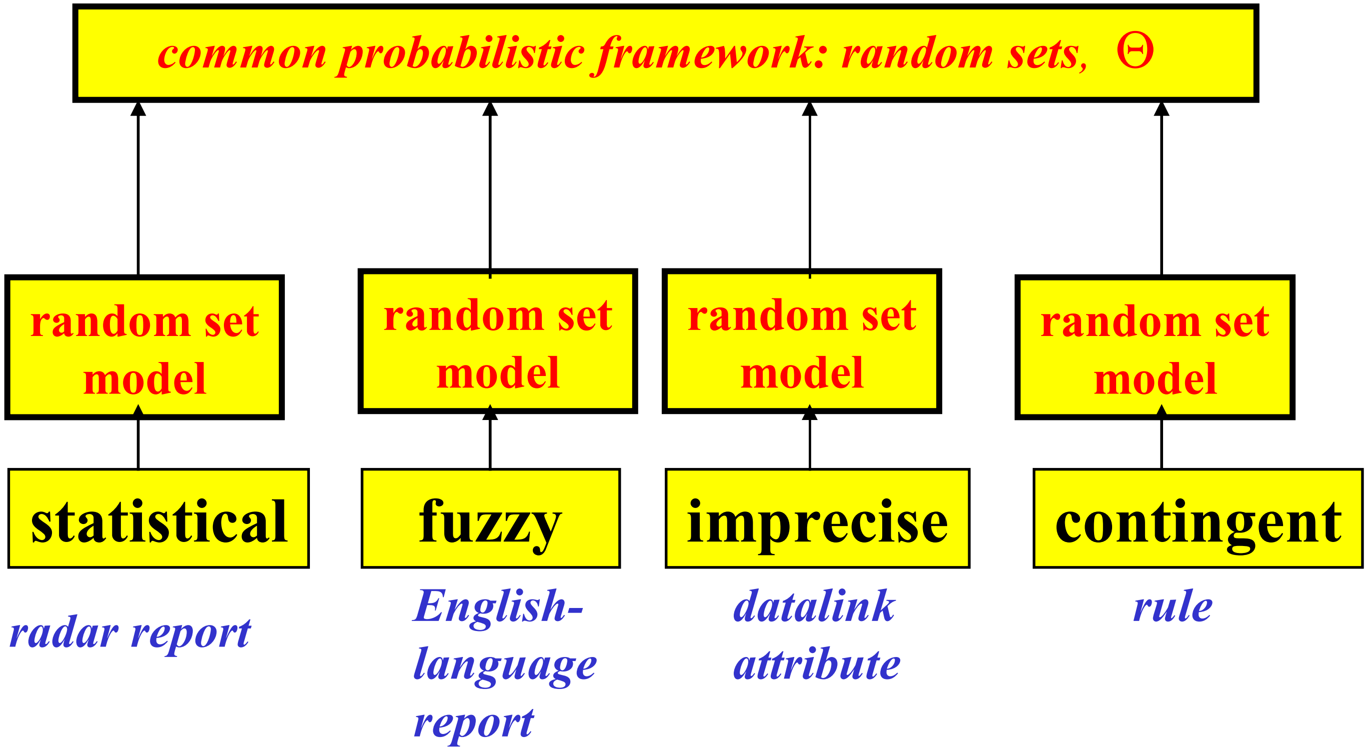
random counting measure

II: Reformulate Multi-Object Problems as Generalized Single-Object Problems



III: Systematic Modeling of Data

probabilistic framework for modeling uncertainties in models



common probabilistic framework: random sets, Θ

random set model

random set model

random set model

random set model

statistical

fuzzy

imprecise

contingent

radar report

English-language report

datalink attribute

rule

IV: Multisource-Multitarget "Statistics 101"

single-sensor/target	*multi-sensor/target*
sensor	global sensor
target	global target
vector sample, $\mathbf{z}$	finite-set sample, Z
vector parameter, $\mathbf{x}$	finite-set parameter, X
derivative, $dp_{\mathbf{Z}}/d\mathbf{z}$	set derivative, $\delta\beta_{\Sigma}/\delta Z$
integral, $\int f(\mathbf{x})\,d\mathbf{x}$	set integral, $\int f(Z)\,\delta Z$
prob.-mass func., $p_{\mathbf{Z}}(S)$	**belief-mass func.,** $\beta_{\Sigma}(S)$
likelihood, $f_{\mathbf{Z}}(\mathbf{z}\|\mathbf{x})$	multitarget likelihood, $f_{\Sigma}(Z\|X)$
prior PDF, $f_0(\mathbf{x})$	multitarget prior PDF, $f_0(X)$
information theory	multitarget information theory
filtering theory	multitarget filtering theory

Almost-parallel Worlds Principle (APWOP)

Almost-Parallel Worlds Principle (APWOP)

"Nearly any single-sensor, single-object concept or algorithm can, in principle, be directly translated into a corresponding multi-sensor, multi-object concept or algorithm."

standard example:

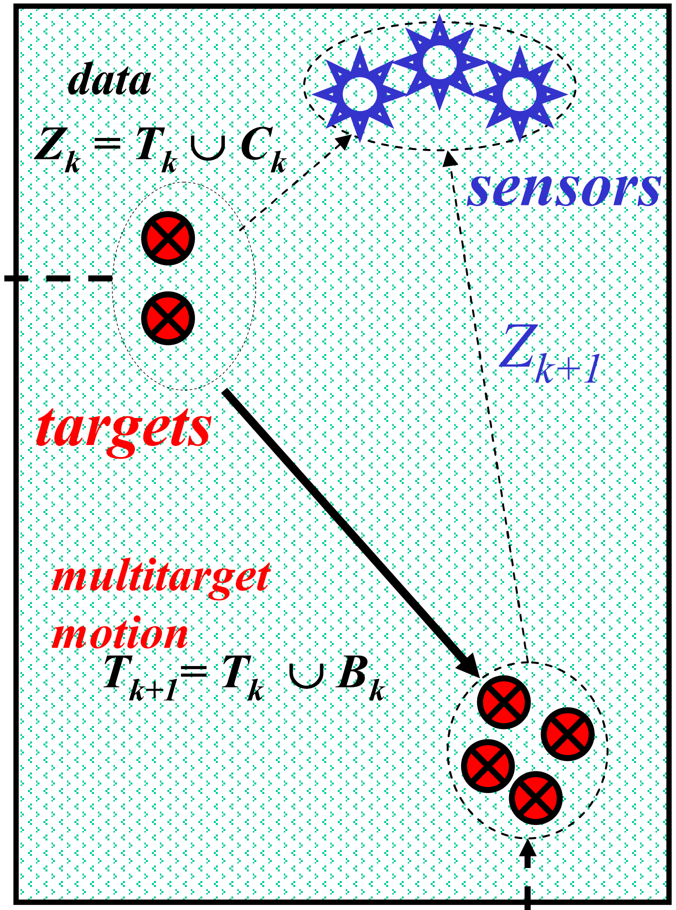
single-target Kullback-Leibler discrimination ➡ ***multitarget Kullback-Leibler discrimination***

$$K(f;g) = \int f(\mathbf{x}) \, log \left[\frac{f(\mathbf{x})}{g(\mathbf{x})} \right] d\mathbf{x}$$

$$K(f;g) = \int f(X) \, log \left[\frac{f(X)}{g(X)} \right] \delta X$$

ordinary posteriors ⇒ multitarget posteriors
ordinary integral ⇒ multitarget "set" integral

Multi-Target Bayes NLF



multitarget Markov motion model

multitarget time prediction

$$f_{k+1/k}(X_{k+1}/Z^{(k)}) = \int f_{k+1/k}(X_{k+1}/X_k)\, f_{k/k}(X_k/Z^{(k)})\ \delta X_k$$

multisensor-multitarget likelihood function

multisensor-multitarget Bayes update

$$f_{k+1/k+1}(X_{k+1}/Z^{(k+1)}) \propto f(Z_{k+1}/X_{k+1})\, f_{k+1/k}(X_{k+1}/Z^{(k)})$$

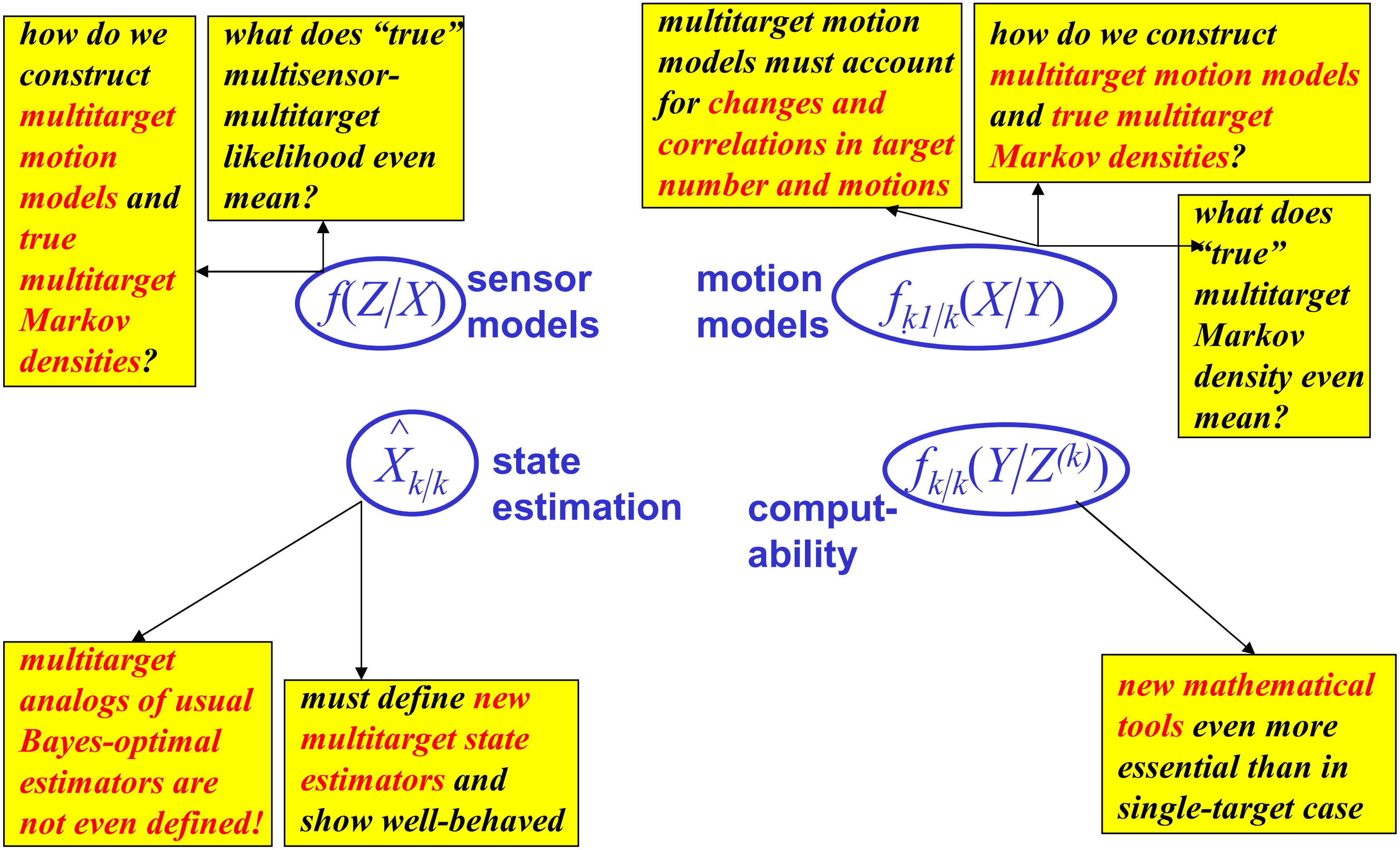
multitarget state estimation

$$\hat{X}_{k+1}$$

Multisensor-Multitarget Bayes Filtering: History

Date	Author(s)	Theoretical Basis
1991	Miller et. al. *("Jump Diffusion")*	Stochastic PDEs
1994	Bethel & Paras	discrete filtering
1994	Mahler *("Finite-Set Statistics")*	FISST
1996	Stone et. Al. *("Unified Data Fusion")*	heuristic
1996	Mahler-Kastella *("Joint Multitarget Probabilities")*	FISST
1997	Portenko et. al.	point processes (random measures)

Basic Issues for Multi-Object Bayes Filtering



how do we construct multitarget motion models and true multitarget Markov densities?

what does "true" multisensor-multitarget likelihood even mean?

$f(Z/X)$ **sensor models**

motion models $f_{k1/k}(X/Y)$

multitarget motion models must account for changes and correlations in target number and motions

how do we construct multitarget motion models and true multitarget Markov densities?

what does "true" multitarget Markov density even mean?

$\hat{X}_{k/k}$ **state estimation**

comput-ability $f_{k/k}(Y/Z^{(k)})$

multitarget analogs of usual Bayes-optimal estimators are not even defined!

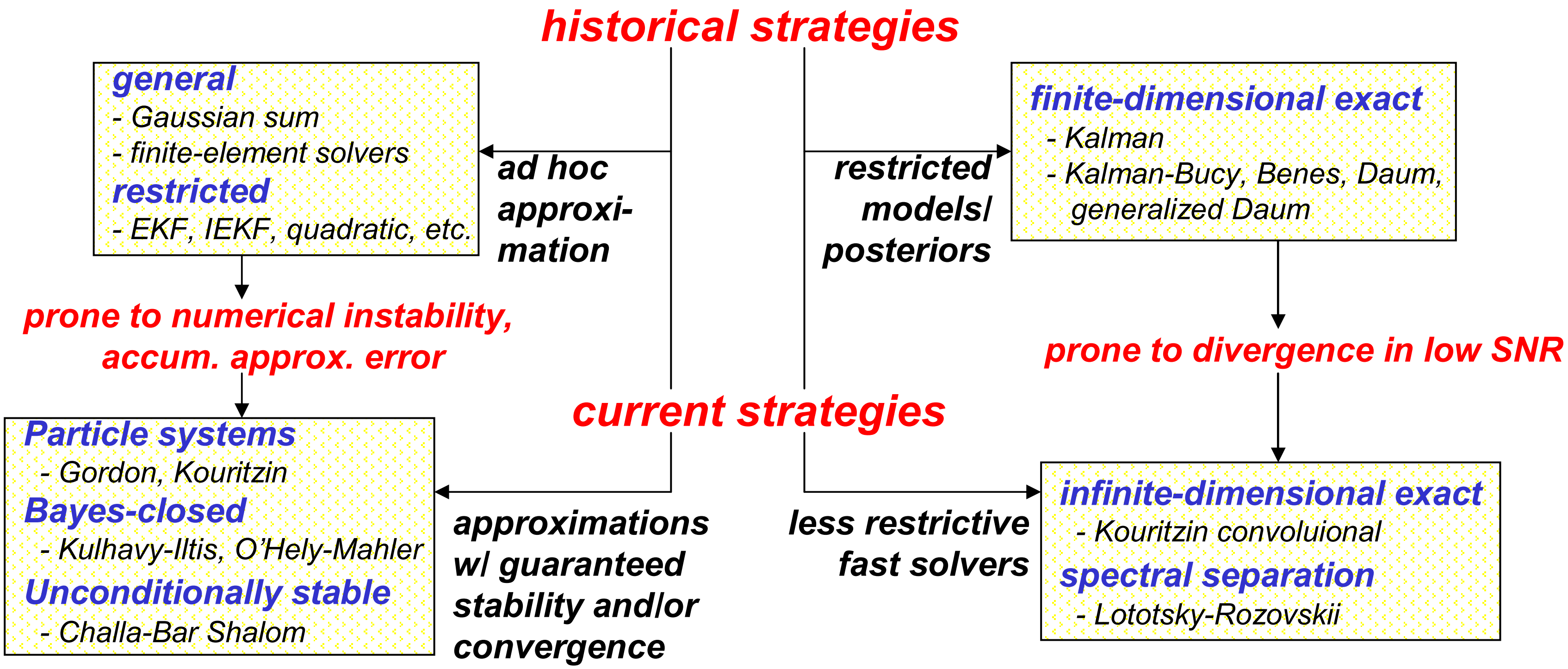
must define new multitarget state estimators and show well-behaved

new mathematical tools even more essential than in single-target case

Computational Problem

computation vs. generality vs. convergence/instability

> ***"Unfortunately, although the manner in which the [a posteriori] density evolves with time and additional measurement data can be described in terms of differential, or difference, equations…these relations are generally very difficult to solve either in closed form or numerically, so that it is usually impossible to determine the a posteriori density for specific applications.*** *- Sorensen & Alspach, 1971*



historical strategies

ad hoc approximation

general
- Gaussian sum
- finite-element solvers

restricted
- EKF, IEKF, quadratic, etc.

prone to numerical instability, accum. approx. error

restricted models/ posteriors

finite-dimensional exact
- Kalman
- Kalman-Bucy, Benes, Daum, generalized Daum

prone to divergence in low SNR

current strategies

approximations w/ guaranteed stability and/or convergence

Particle systems
- Gordon, Kouritzin

Bayes-closed
- Kulhavy-Iltis, O'Hely-Mahler

Unconditionally stable
- Challa-Bar Shalom

less restrictive fast solvers

infinite-dimensional exact
- Kouritzin convoluional

spectral separation
- Lototsky-Rozovskii

Multi-Object Integral and Differential Calculus

(computable using "turn-the-crank" formulas)

- **Set integral:**

$$\int f(X)\delta X = \sum_{k=0}^{\infty} \frac{1}{k!} \int f(\{\mathbf{x}_1,\ldots,\mathbf{x}_k\})\, d\mathbf{x}_1 \cdots d\mathbf{x}_k$$

- **Set derivative:**

$$f_\Sigma(Z) = \boxed{\frac{\delta\beta_\Sigma}{\delta Z}}(\emptyset)$$

multi-object density

set derivative

belief-mass function

Belief-Mass Functions and Set Derivatives

- ***Probability generating functional*** **of random track-set** $\Xi_{k/k}$**:**

$$G_{k/k}[h] = \mathrm{E}[\Pi_{\mathbf{x}\in\Xi}\, h(\mathbf{x})]$$

- ***Functional derivative*** **of** $G_{k/k}$**:**

$$\frac{\partial G_{k/k}}{\partial \mathbf{x}}[h] = \lim_{\varepsilon\to 0} \frac{G_{k/k}[h+\varepsilon\delta_{\mathbf{x}}] - G_{k/k}[h]}{\varepsilon}$$

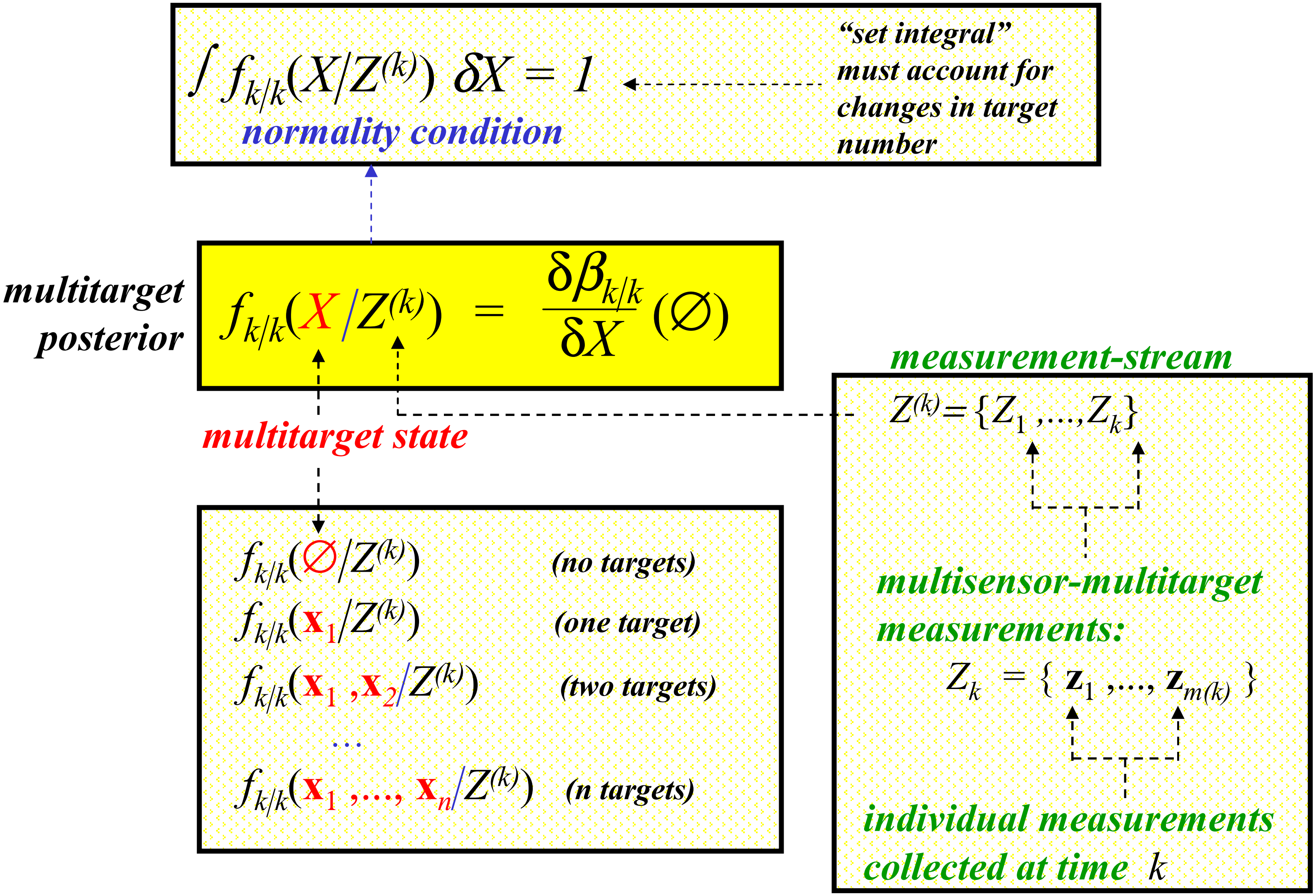
- ***Belief-mass function*** **of** $\Xi_{k/k}$

$$\beta_{k/k}(S) = G_{k/k}[\mathbf{1}_S] = \Pr(\Xi \subseteq S) \equiv \textit{prob-mass function for Mathéron topology}$$

- ***Set derivative*** **of** $\beta_{k/k}$**:**

$$\frac{\delta\beta_{k/k}}{\delta X}(S) = \frac{\partial^n G_{k/k}}{\partial \mathbf{x}_1 \cdots \partial \mathbf{x}_n}[\mathbf{1}_S] \qquad X=\{\mathbf{x}_1,\ldots,\mathbf{x}_n\}$$

Multi-Object Posterior Density Functions



$$\int f_{k|k}(X|Z^{(k)})\ \delta X = 1$$

normality condition

"set integral" must account for changes in target number

multitarget posterior

$$f_{k|k}(X\,|Z^{(k)}) = \frac{\delta\beta_{k|k}}{\delta X}(\emptyset)$$

multitarget state

$$f_{k|k}(\emptyset|Z^{(k)})$$ *(no targets)*

$$f_{k|k}(\mathbf{x}_1|Z^{(k)})$$ *(one target)*

$$f_{k|k}(\mathbf{x}_1,\mathbf{x}_2|Z^{(k)})$$ *(two targets)*

$$\ldots$$

$$f_{k|k}(\mathbf{x}_1,\ldots,\mathbf{x}_n|Z^{(k)})$$ *(n targets)*

measurement-stream

$$Z^{(k)} = \{Z_1,\ldots,Z_k\}$$

multisensor-multitarget measurements:

$$Z_k = \{\mathbf{z}_1,\ldots,\mathbf{z}_{m(k)}\}$$

individual measurements collected at time k

True Multi-Object Likelihoods & Markov Densities

OBSERVATIONS

measurement model

all observations (object or clutter)		observation due to targets (if present)		observations due to clutter generators
Z_k	$=$	T_k	$\cup$	C_k

belief-mass function

$$\beta_k(S/X) \;=\; \Pr(Z_k \subseteq S)$$

= probability that all observations lie within S

constructed likelihood function

$$f_k(Z/X) = \frac{\delta\beta_k}{\delta Z}(\emptyset\,/X)$$

= likelihood of seeing observation-set Z*, given target group with state-set* X

MOTION

motion model

target states at new time-step		target states of surviving old targets		target states of new targets
T_{k+1}	$=$	T_k	$\cup$	B_k

belief-mass function

$$\beta_{k+1/k}(S/X) \;=\; \Pr(T_{k+1} \subseteq S)$$

= probability that all new targets lie within S

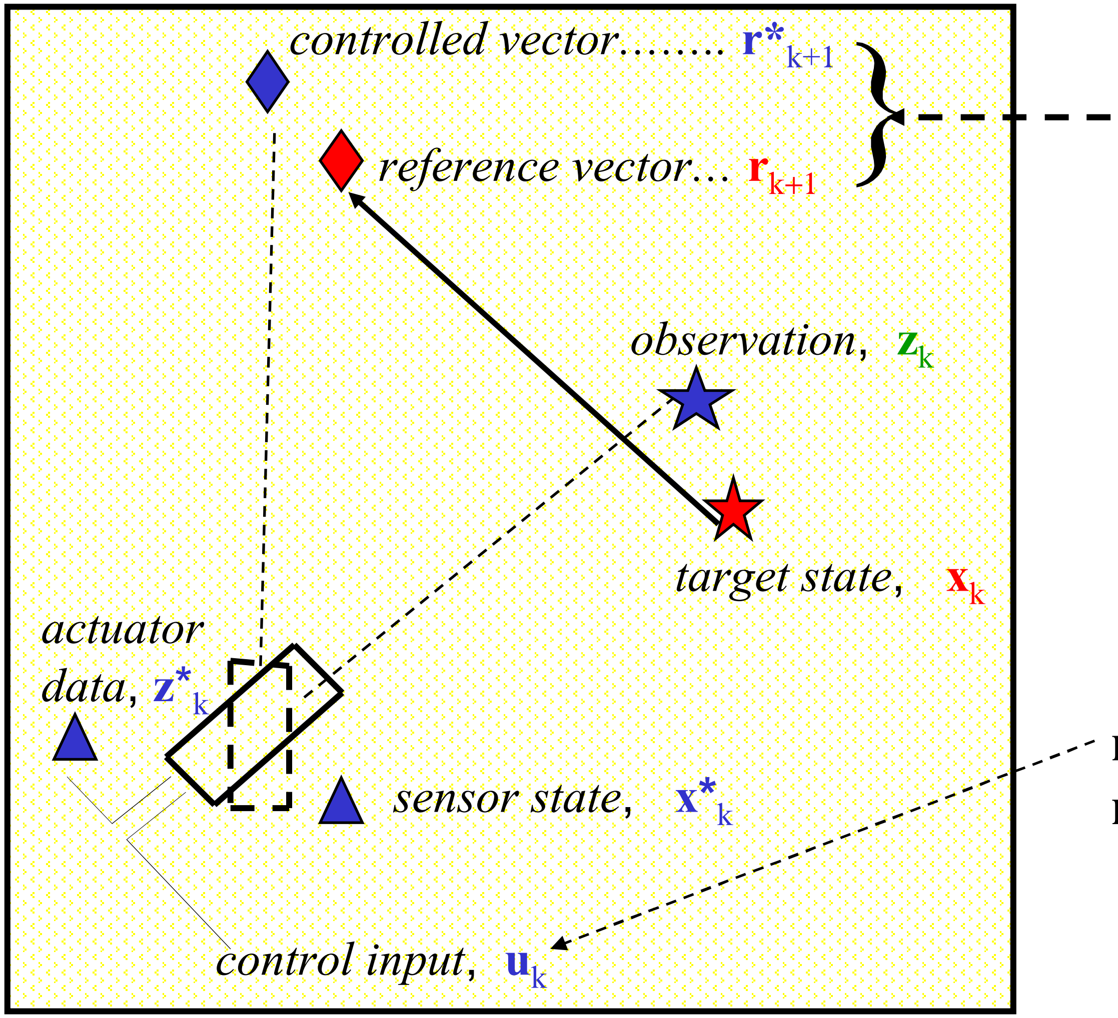
constructed Markov transition density

$$f_{k+1/k}(Y/X) = \frac{\delta\beta_{k+1/k}}{\delta Y}(\emptyset/X)$$

= likelihood of seeing target-set Y*, given that targets previously had state-set* X

Multisource-Multitarget Sensor Management

Single-Sensor, Single Target Control



controlled vector…….. $\mathbf{r}^*_{k+1}$

reference vector… $\mathbf{r}_{k+1}$

minimize distance

(e.g. Mahalanobis)

observation, $\mathbf{z}_k$

target state, $\mathbf{x}_k$

actuator data, $\mathbf{z}^*_k$

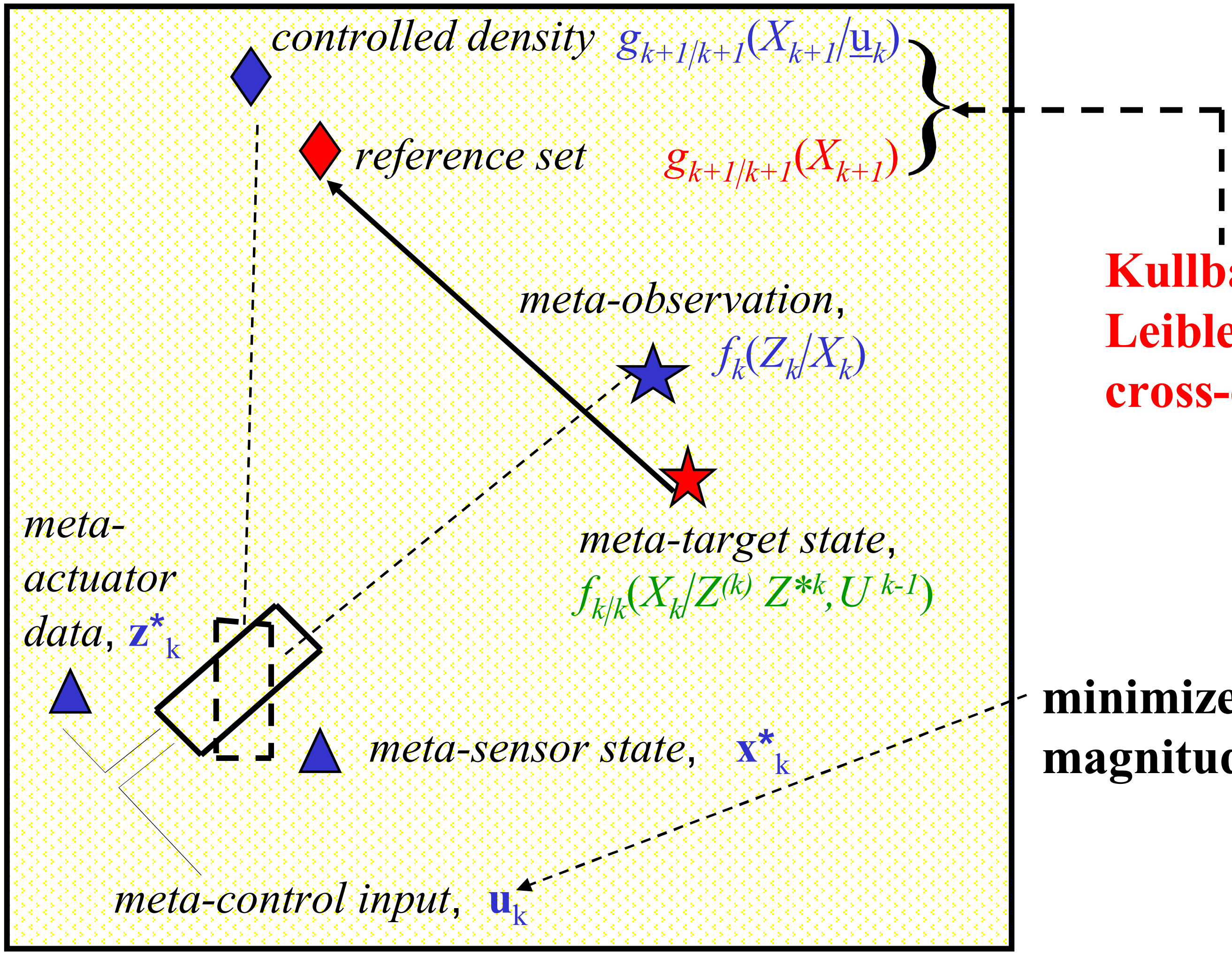
sensor state, $\mathbf{x}^*_k$

minimize magnitude

control input, $\mathbf{u}_k$

Information-Based Multisensor-Multitarget Control

R. Mahler, "Global Optimal Sensor Allocation," *Proc. Ninth Nat'l Symp. on Sensor Fusion*, Mar. 1996



controlled density $g_{k+1|k+1}(X_{k+1}/\underline{u}_k)$

reference set $g_{k+1|k+1}(X_{k+1})$

Kullback-Leibler cross-entropy

meta-observation, $f_k(Z_k/X_k)$

meta-target state, $f_{k|k}(X_k/Z^{(k)} Z^{*k}, U^{k-1})$

meta-actuator data, $\mathbf{z}^*_k$

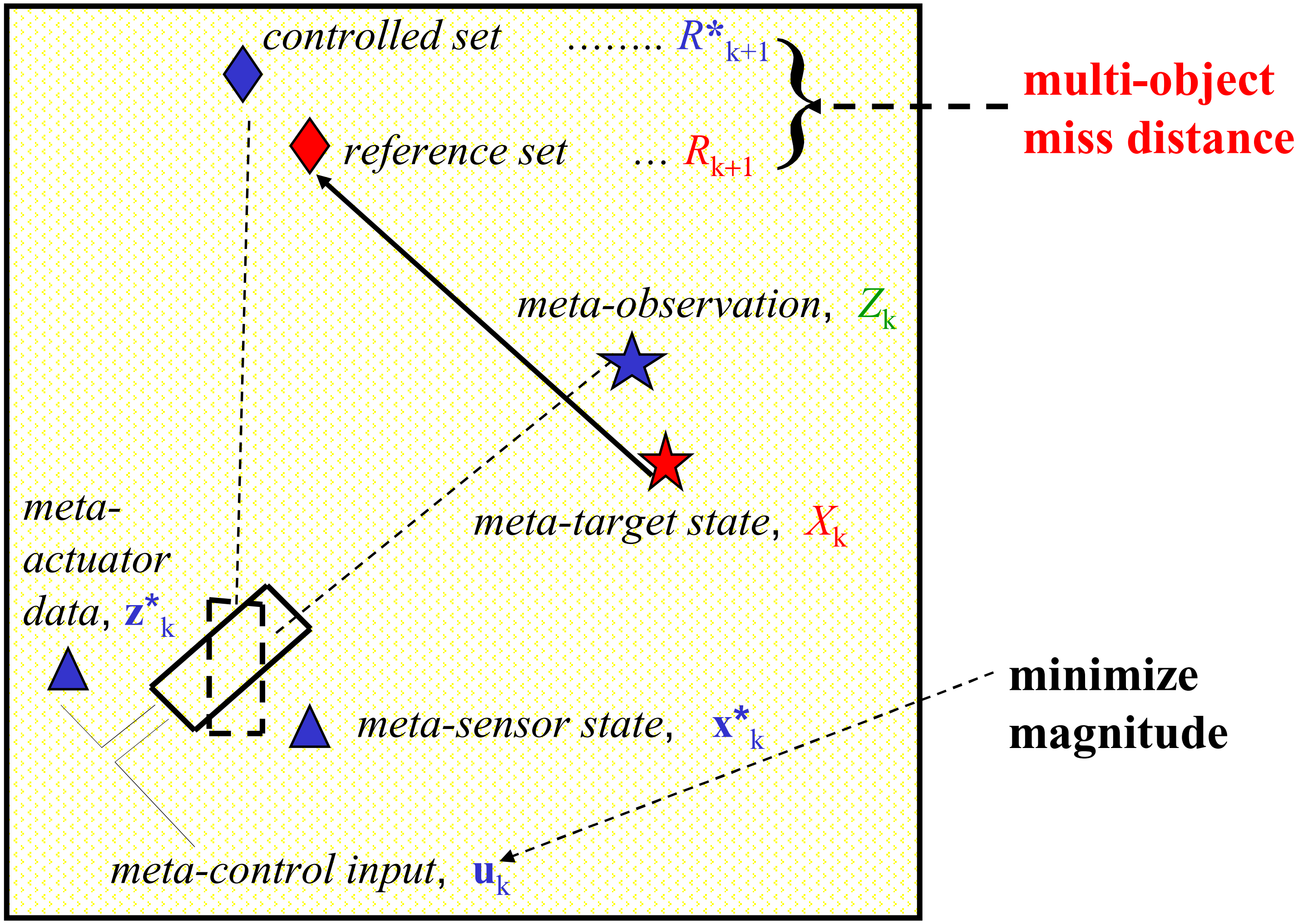
minimize magnitude

meta-sensor state, $\mathbf{x}^*_k$

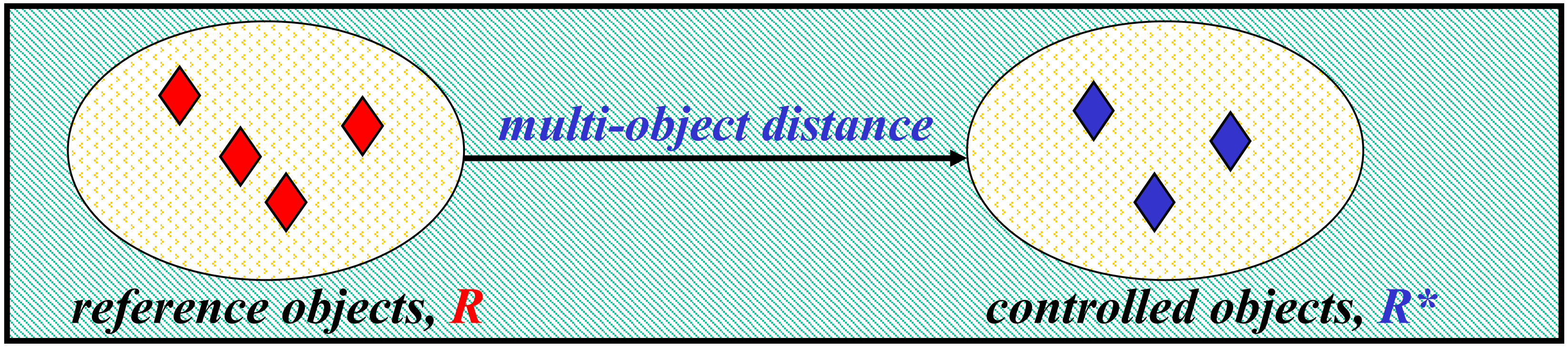
meta-control input, $\mathbf{u}_k$

however, computational tractability is doubtful except in special cases

Direct Approach to Multisensor-Multitarget Control



Examples: Multi-Object Distance Metrics



multi-object distance

reference objects, R

controlled objects, R*

Hausdorff distance

$$d_H(R,R^*) = \max\{\ \max_{\mathrm{r}\in R} \min_{\mathbf{r}^*\in R^*} d(\mathbf{r},\mathbf{r}^*),\ \ \max_{\mathrm{r}^*\in R^*} \min_{\mathbf{r}\in R} d(\mathbf{r},\mathbf{r}^*)\ \}$$

Wasserstein-Mallows distance

$$d_W(R,R^*) \ = \ \max_C \sum_{i=1}^{|R|}\sum_{j=1}^{|R^*|} C_{i,j}\, d(\mathbf{r}_i, \mathbf{r}^*_j), \ \ \textbf{\textit{where}} \ \ \sum_{i=1}^{|R|} C_{i,j} = |R^*|^{-1}, \ \ \sum_{j=1}^{|R^*|} C_{i,j} = |R|^{-1}$$

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