Conference on Cooperative Control and Optimization Open Discussion

13 Nov 01

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Outline of Talk

- Recap of Last Year's Opens Discussion
- A Taxonomy for Collective Systems
- Implications

Dec 00 Discussion Recap

Going in definition of a cooperative system

Multiple dynamic entities that share information, tasks and possibly constraints to accomplish a common , though perhaps not singular, objective.

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- Define sharing.
- Should definition attempt to encapsulate the following:

1. Total cost added (for communication) must provide a greater increase in expected system effectiveness than like cost being used to add additional (non-cooperating) agents.
2. Performance lower bound of a cooperative system as communication degrades should never be worse than the performance of the same agents without cooperation.
3. Cooperating robotic agents are meant to serve humans. Hence cooperative systems must accommodate humans in-the-loop at some level of abstraction for annunciation and control. Efficiency of the cooperative system must not suffer as a result.

If so, is there a concise or mathematical expression for this?

- Stress that there may be time and context specific objectives.
- Decomposition/Decoupling of Cooperative Systems: is there a minimal system that can only be defined in terms of cooperation? If so, this defines cooperation.
- Along the same lines, consider that cooperation is just one level of integration between 2 or more systems, the others being coordination/complementation and combination.
- Example: accidental (Serendipity) vs. intentional cooperation is not truly cooperative.

Dec 00 Open Discussion Recap

What questions should our research answer?

- How are information and authority related?
- What information should be sent and to whom?
- When are informationally decentralized, rule-based “emergent” approaches sufficient or optimal?
- When are cooperative systems stable?
- How do we manage uncertainty?
- How do you test cooperative systems (from simulation building up to hardware)?
What are the metrics for cooperative system performance?

Collective Systems Taxonomy

A collective system is a collection of at least 2 dynamical entities, each with a set of individually admissible decision policies.

Every decision policy may be a function of observations of the collective system state and necessarily modifies the decision maker's state in a nontrivial fashion in finite time.

Mathematically,

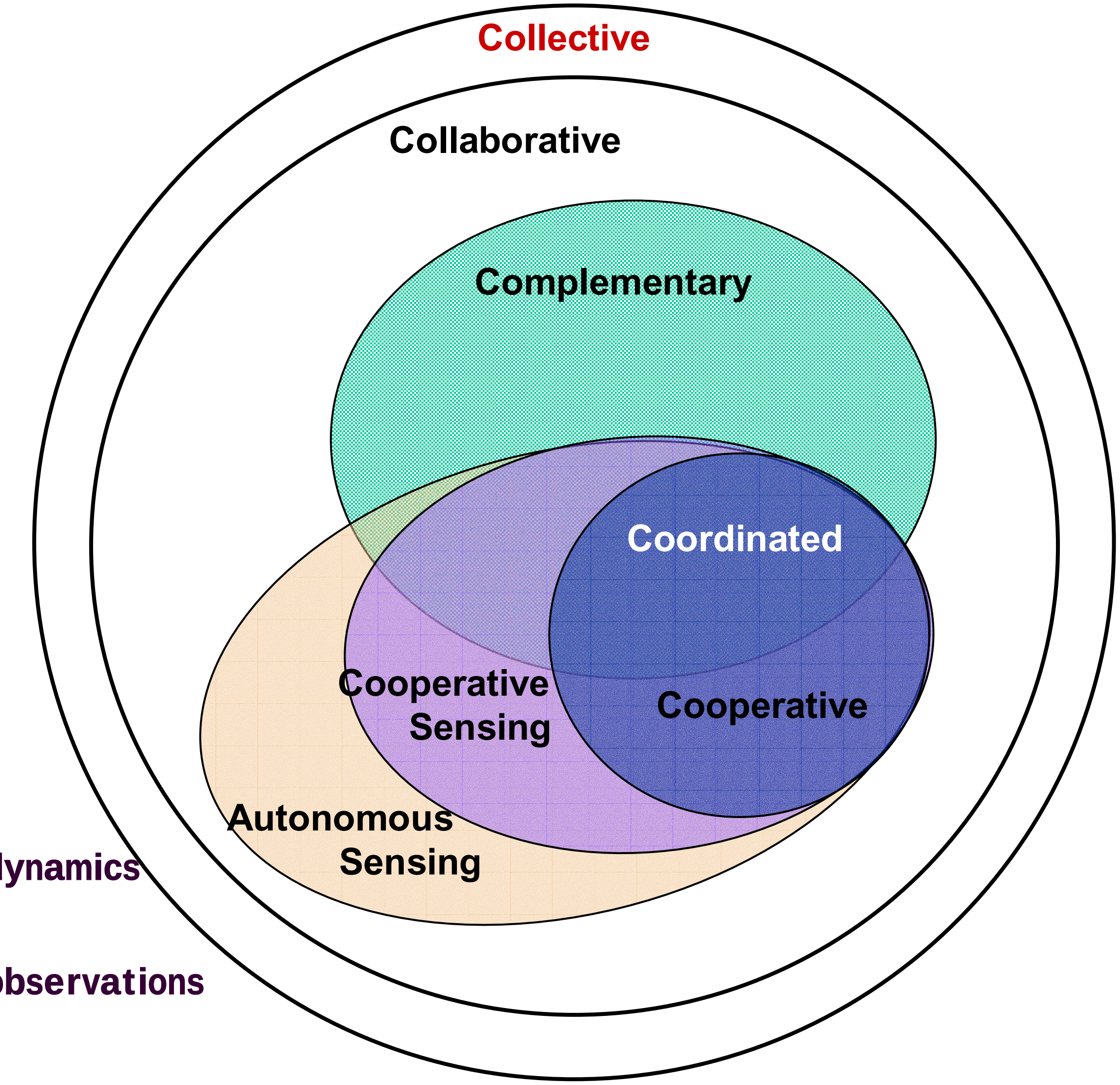
$x(t) = [x_1(t), \ldots, x_N(t)]$, states

$u(t) = [u_1(t), \ldots, u_N(t)]$, decisions

$dx_i(t)/dt = f_i(x_i(t), u_i(t), t)$, Collective dynamics

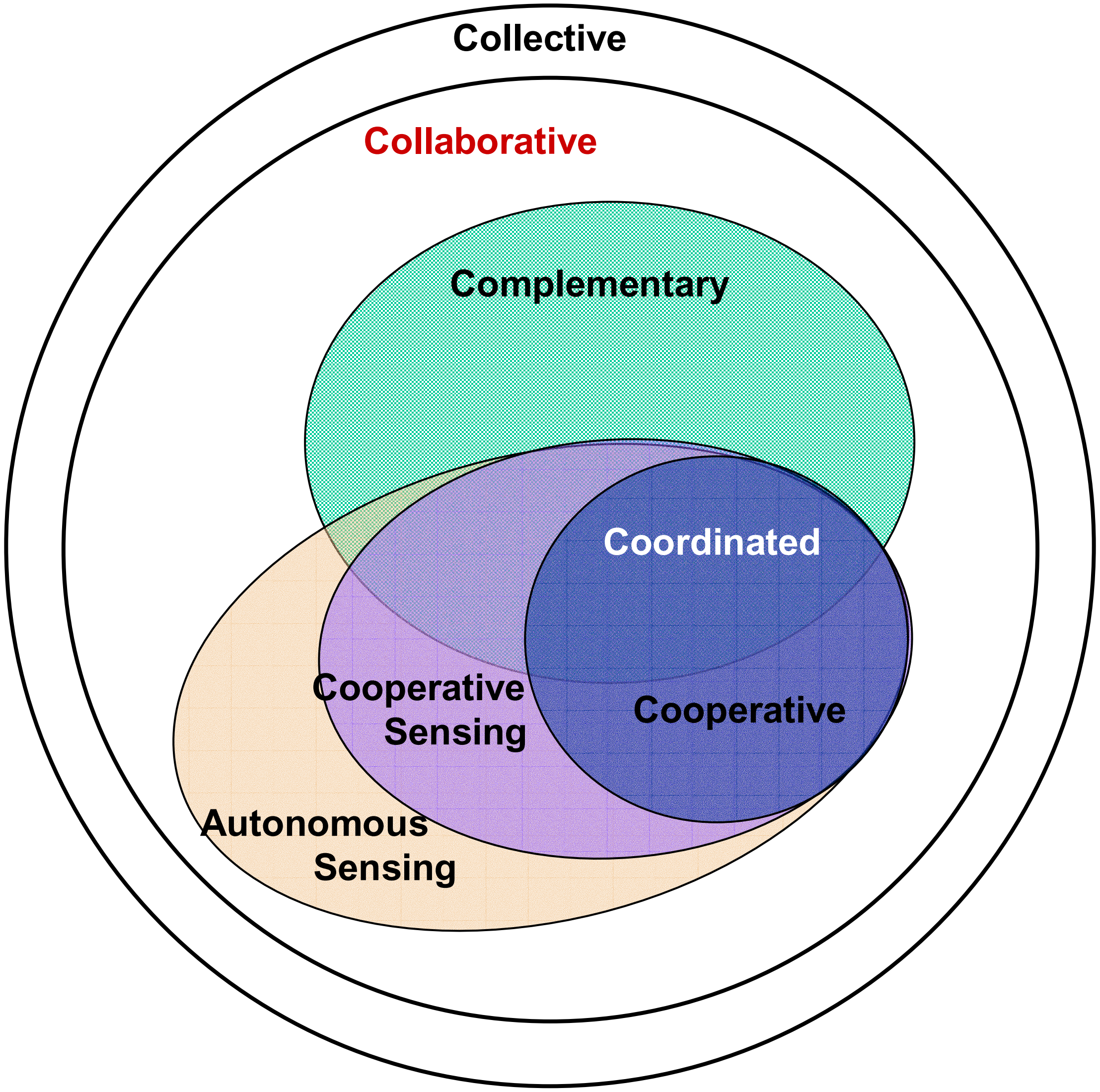
$z_i(t) = h_i(x_i(t), t)$, $i = 1, \ldots, N$, $N \geq 2$

Collective observations



Collective Systems Taxonomy

A collective system, such that all decision makers participate in a common scalar payoff J(x, u, t), is termed collaborative.



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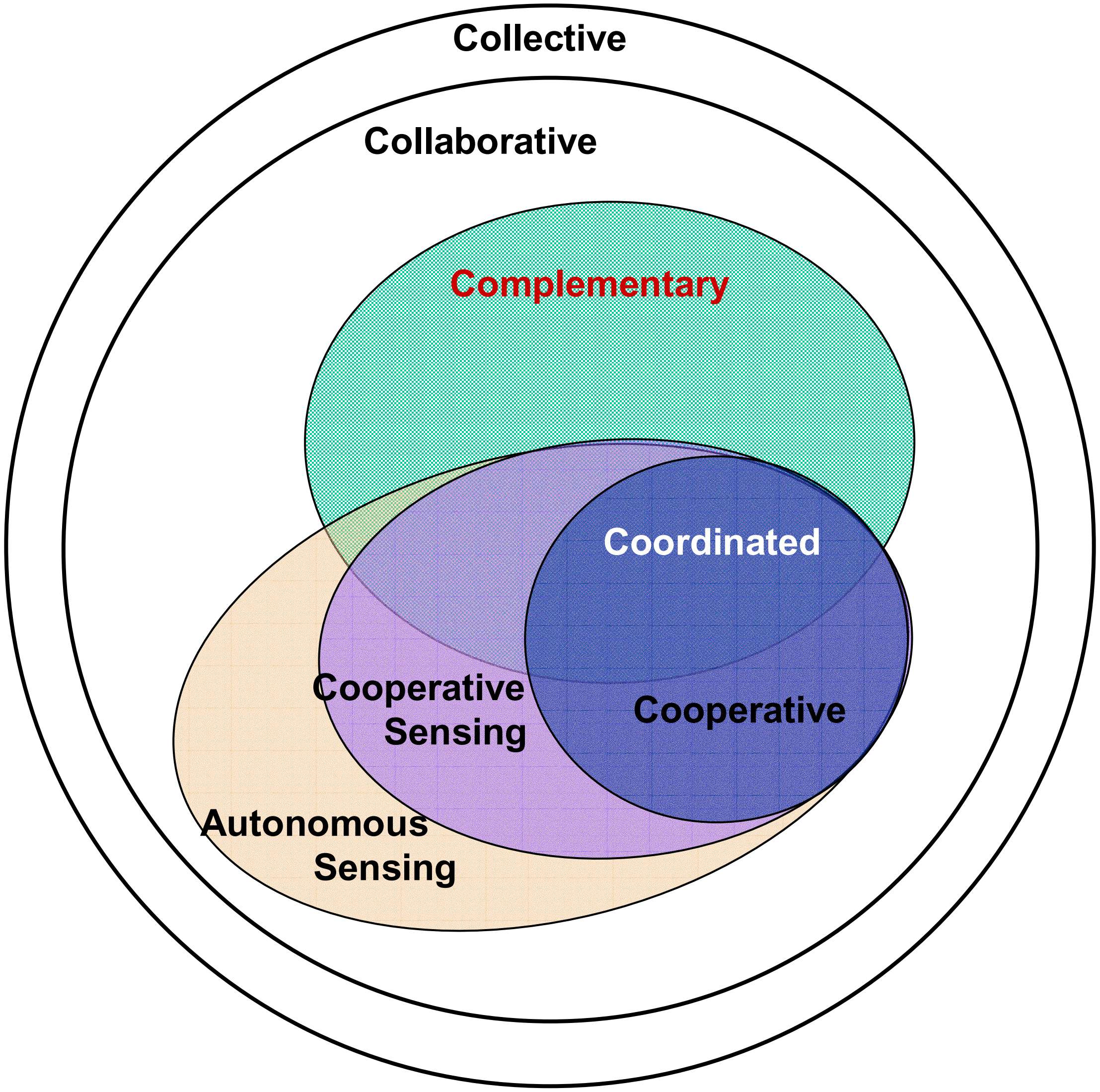
Consider the decisions

$u(t)= [u_1(t),\ldots, u_N(t)]$,
$u_i(t) = \gamma_i(z_i(t),t)$: $\gamma_i \in \Gamma_i$.
If for all $\gamma_i \in \Gamma_i$ there exists an admissible $\gamma_j \in \Gamma_j$ for all $j \neq i$, then $u_i(t)$ is called a complementary decision.

If u_i are complementary for all $i=1,\ldots,N$, the system of decisions $u(t)$ is termed complementary.



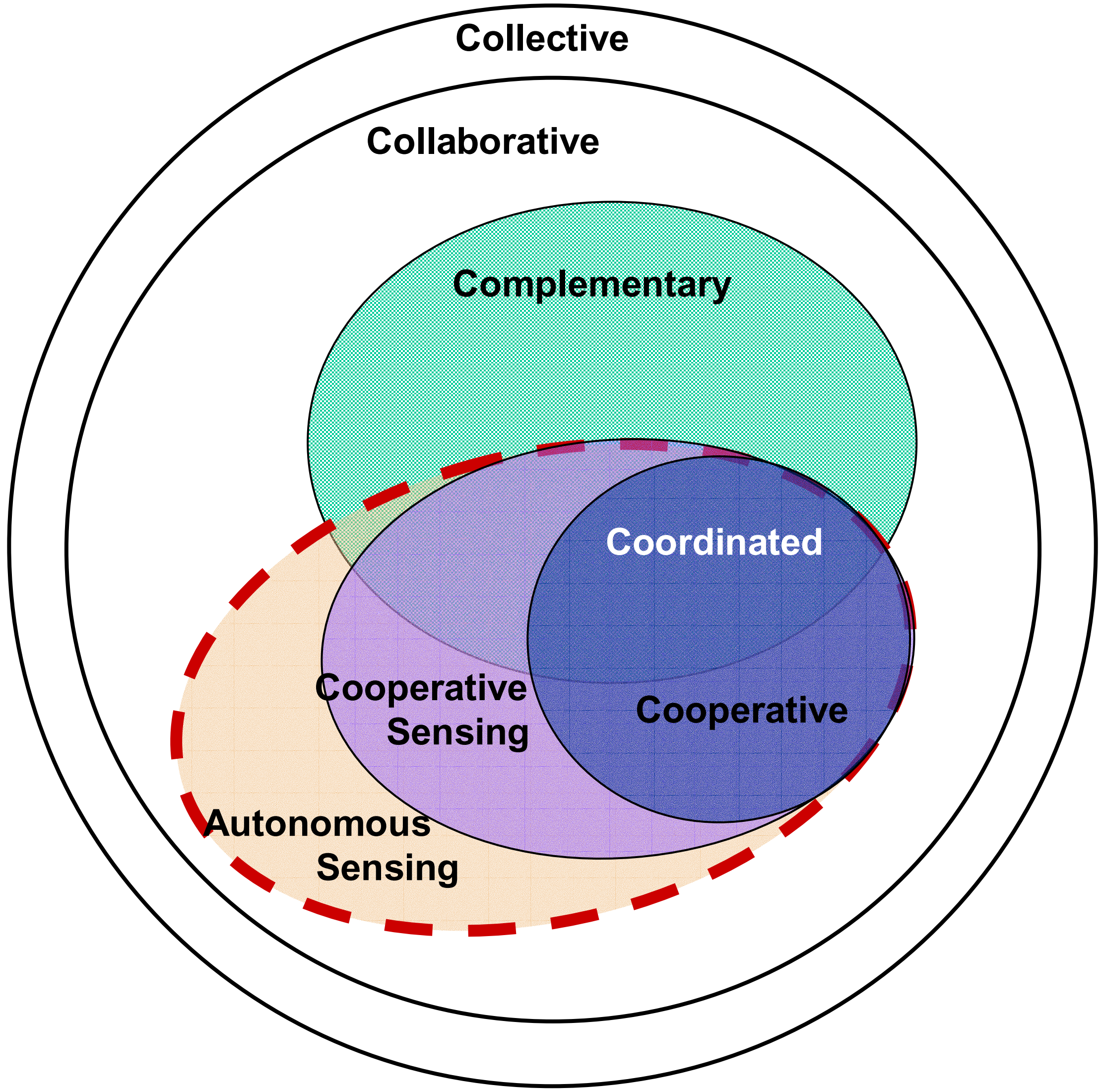
Collective Systems Taxonomy

Consider the collaborative entities with decisions
$u(t) = [u_1(t), \ldots, u_N(t)]$,
$u_i(t) = \gamma_i(y_i(t),t)$: $\gamma_i \in \Gamma_i$ $i=1,\ldots,N$,

where $y_i(t)$ is an m_i-vector of <u>measurements</u> at entity i.

If $y_i(t) = h_{ij}(x_j(t),t) \neq 0$ for $j \neq i$, then entity i is said to <u>sense</u> entity j.

An entity that senses at least one other entity is termed <u>sensing</u>.



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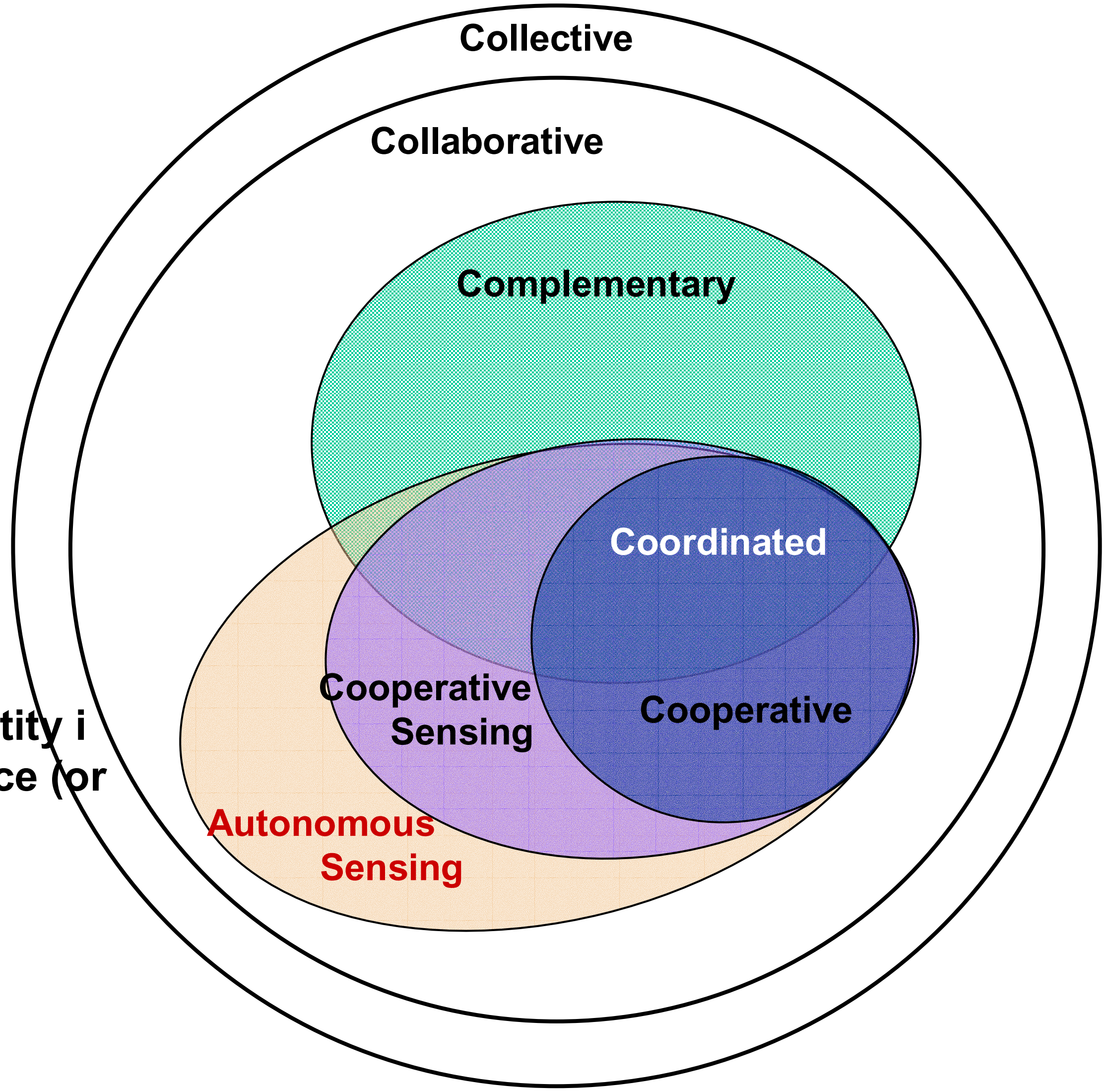
Sensing may take one of 2 fundamental forms.

1. <u>Autonomous sensing</u> occurs if entity i senses entity j without any assistance (or perhaps knowledge) from entity j.



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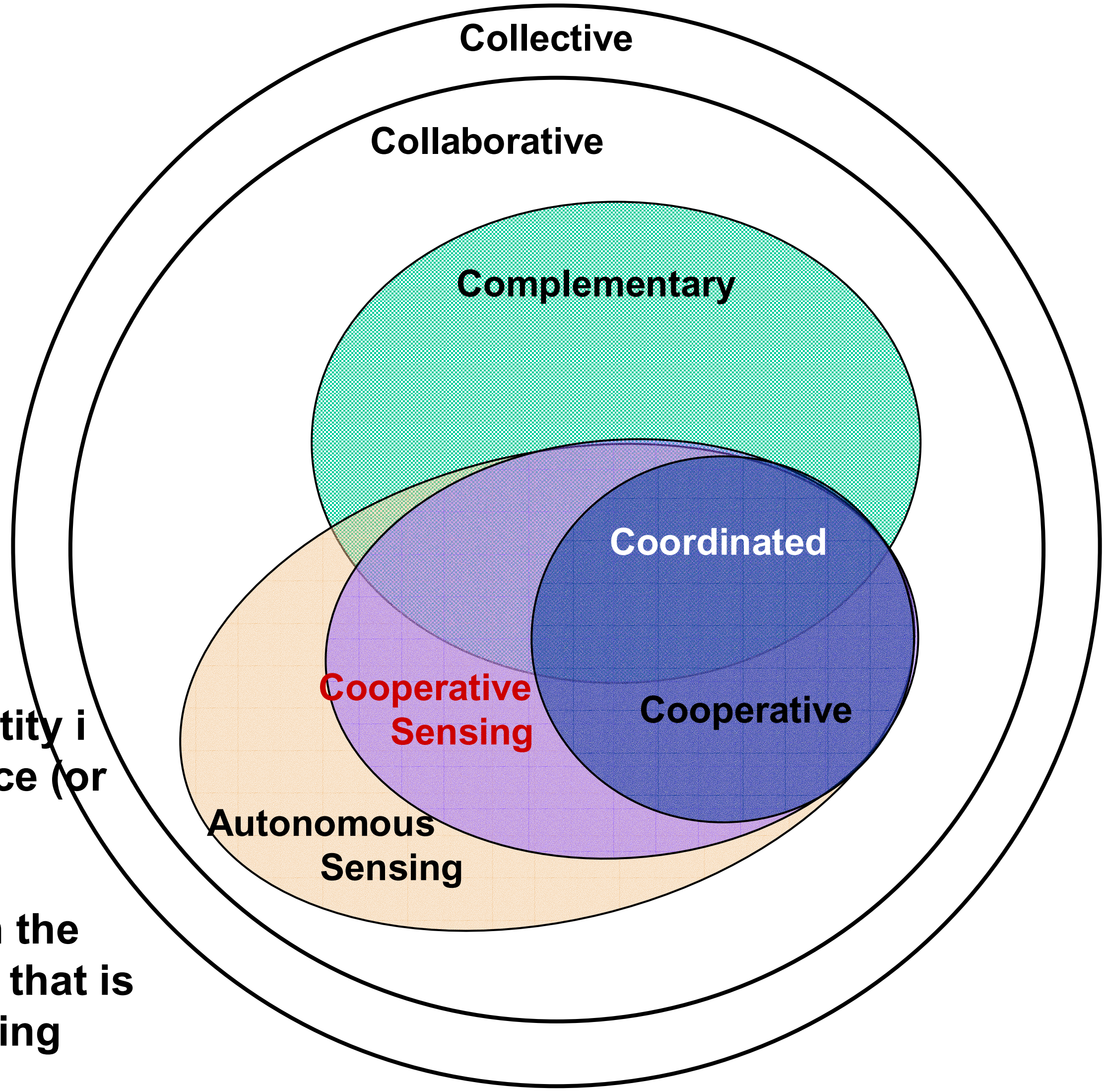
Sensing may take one of 2 fundamental forms.

1. <u>Autonomous sensing</u> occurs if entity i senses entity j without any assistance (or perhaps knowledge) from entity j.

2. <u>Cooperative sensing</u> occurs when the observed entity performs some task that is solely designed to assist the observing entity in determining $h_{ij}(x_j(t),t)$.



Collective Systems Taxonomy

Consider the measurements of entity j excluding those sourced at entity i

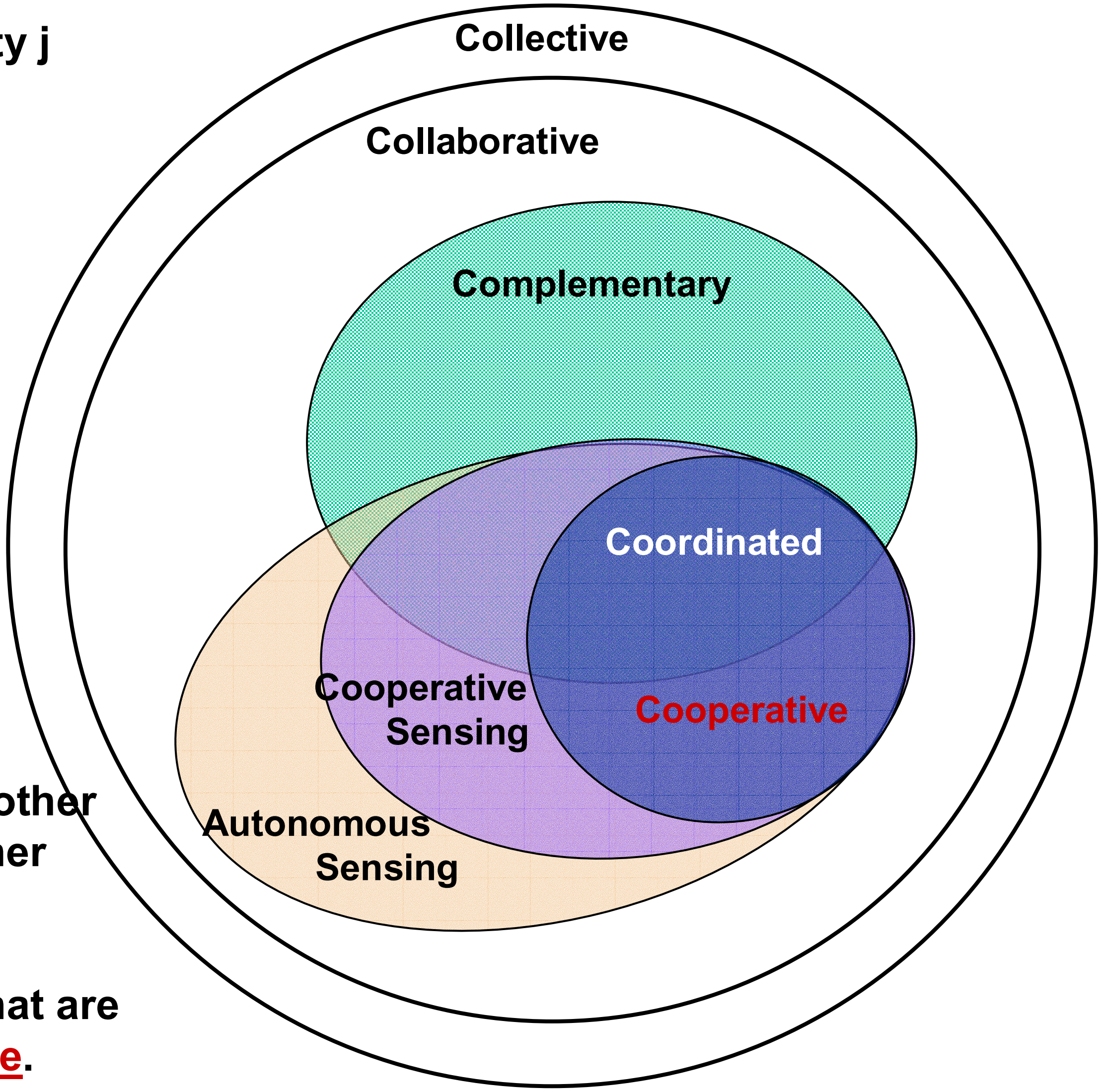
**$y_{j\backslash i}(t) = [\, h_{jk}(x_k(t),t),$ for all $k \in H_j \,]$,
with $H_j \in [1,2, \ldots, j-1, j+1, \ldots, N]$**

Define $z_i(t) = d_{ij}(y_{j\backslash i}(t),t) \neq 0$ for $j \neq i$ as a <u>communicated message</u> of the state of entity j as received by entity i.

d_{ij} denotes the transmission function from entity i to j and models the pertubations due to the transmitter, channel, and receiver.

An entity that receives at least one other communicated message from another entity is termed <u>communicating</u>.

A collaborative system of entities that are communicating is called <u>cooperative</u>.

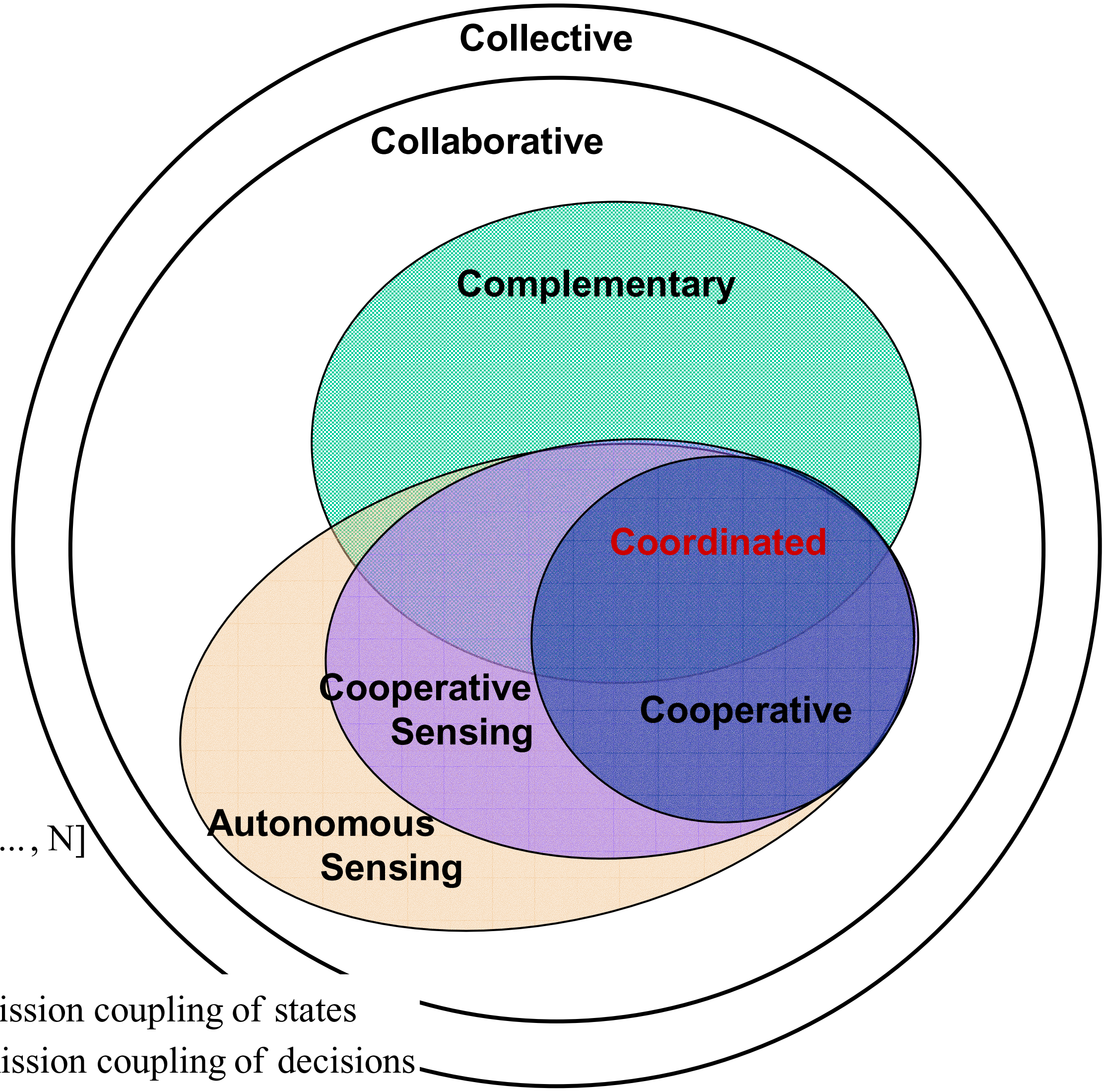


Implies communications graph must be connected for cooperation.

Collective Systems Taxonomy

A complementary system that is not coordinated is easily recognized as having the properties widely associated with emergent systems, that is, a complementary rule set without the benefit of communication.

In summary, the system equations are:



For $i = 1, 2, \ldots, N, \ N \geq 2$

$$\text{minimize } J_i\left(x_i(t), u_i(t), t\right)$$

$$\dot{x}_i(t) = f_i\left(x_i(t), u_i(t), t\right),$$

$$y_i(t) = \left[h_{ij}\left(x_j(t), t\right) \ \forall j \in \mathbf{H}_i\right] \quad \text{where } \mathbf{H}_i \subseteq [1, 2, \ldots, \text{N}]$$

$$y_{j\backslash i}(t) = y_j(t): \ \mathbf{H}_j = \{\mathbf{H}_j - i\}$$

$$z_i(t) = \left[d_{ij}\left(y_{j\backslash i}(t), t\right) \ \forall j \in \mathbf{D}_i, u_k(t), \ \forall k \in \mathbf{C}_i\right]$$

$$\text{where } \mathbf{D}_i \subseteq [1, 2, \ldots, i-1, i+1, \ldots, \text{N}] \text{ xmission coupling of states}$$

$$\text{and } \mathbf{C}_i \subseteq [1, 2, \ldots, i-1, i+1, \ldots, \text{N}] \text{ xmission coupling of decisions}$$

$$u_i(t) = \gamma_i\left(y_i(t), z_i(t), t\right), \quad \gamma_i \in \Gamma_i \neq \varnothing$$

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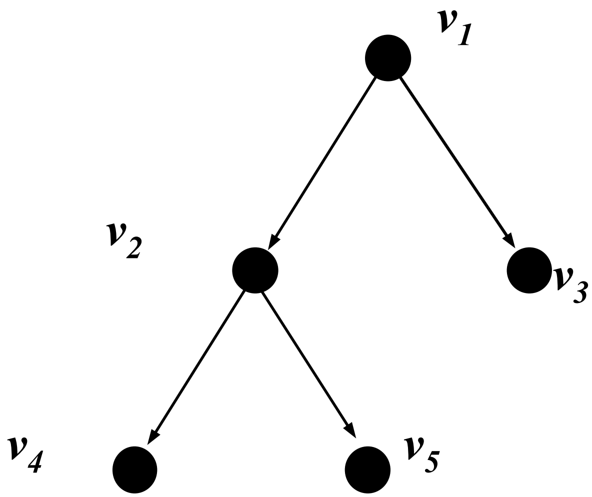
Precedence

- Start with Ho-Chu 72 framework of precedence



Precedence Structure

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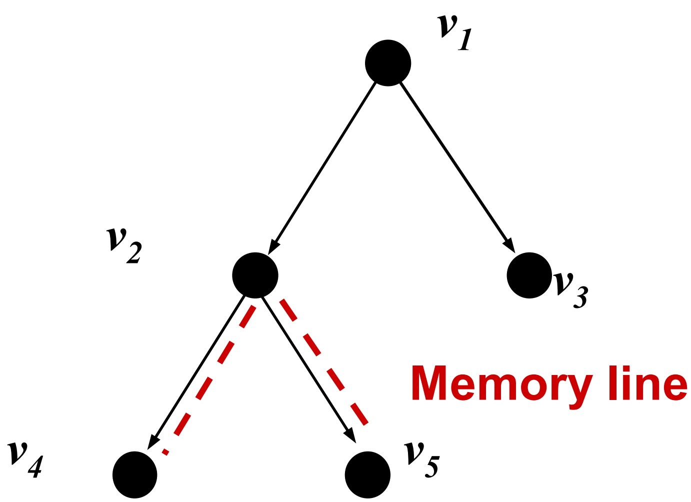

Precedence Structure

[Ho-Chu 72] Partially nested condition satisfied if j precedent to i implies z_i contains z_j where

$$z_i = H_i \xi + \sum_{j=1}^{N} D_{ij} u_j, \; i = 1, \ldots, N$$

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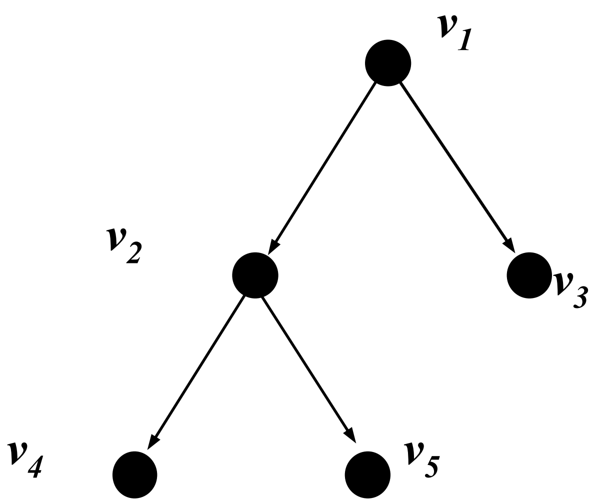
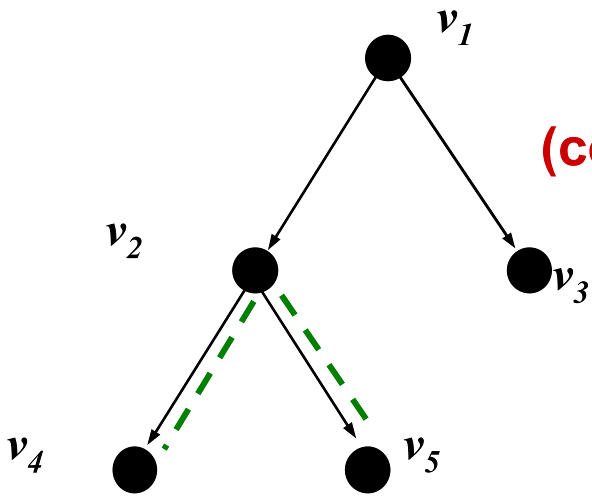


Precedence Structure

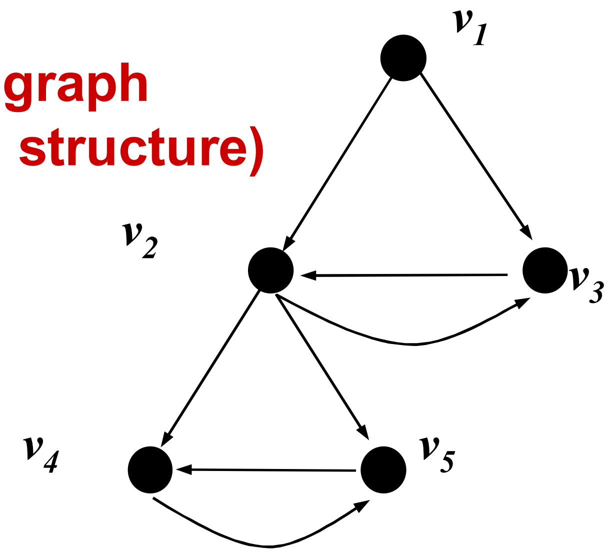
Problem: “non-nested” (finite memory) precedence is in practice very hard to develop control policies for, even for systems with linear states and quadratic cost.

Precedence

- Start with Ho-Chu 72 framework of precedence
- Adopt terminology of wireless networking community



Multigraph (coupling structure)



Precedence Structure **Communication Structure**

$$z_i(t) = \left[d_{ij}\left(y_{j\backslash i}(t), t \right)\ \forall j \in \mathbf{D}_i, u_k(t),\ \forall k \in \mathbf{C}_i \right]$$

Strong nesting condition

$=$ where $\mathbf{D}_i \subseteq [1, 2, \dots, i-1, i+1, \dots, \mathrm{N}]$ xmission coupling of states

$=$ and $\mathbf{C}_i \subseteq [1, 2, \dots, i-1, i+1, \dots, \mathrm{N}]$ xmission coupling of decisions

Precedence

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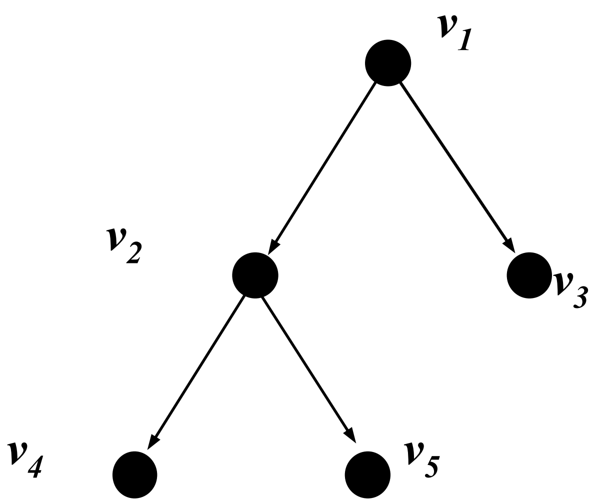
Send decisions
Directly to all decendants



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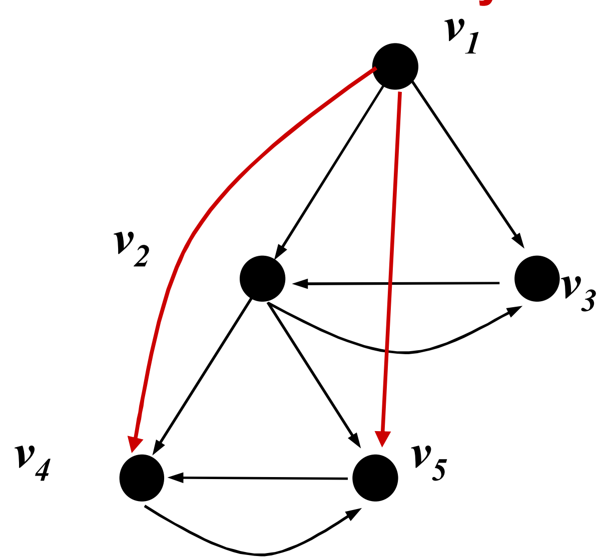

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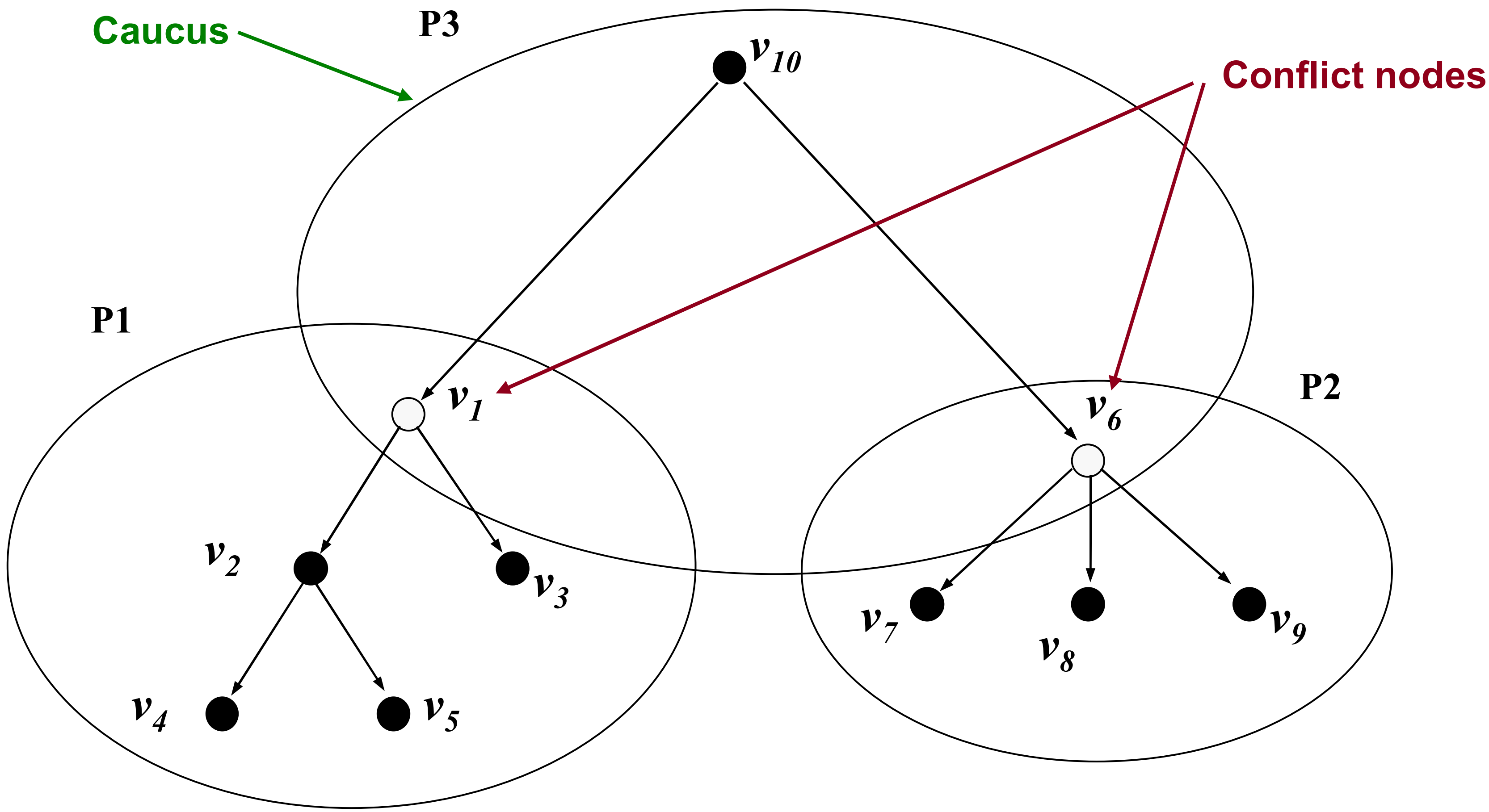
Weak nesting condition

Multiple Objectives, Caucuses and Hierarchy

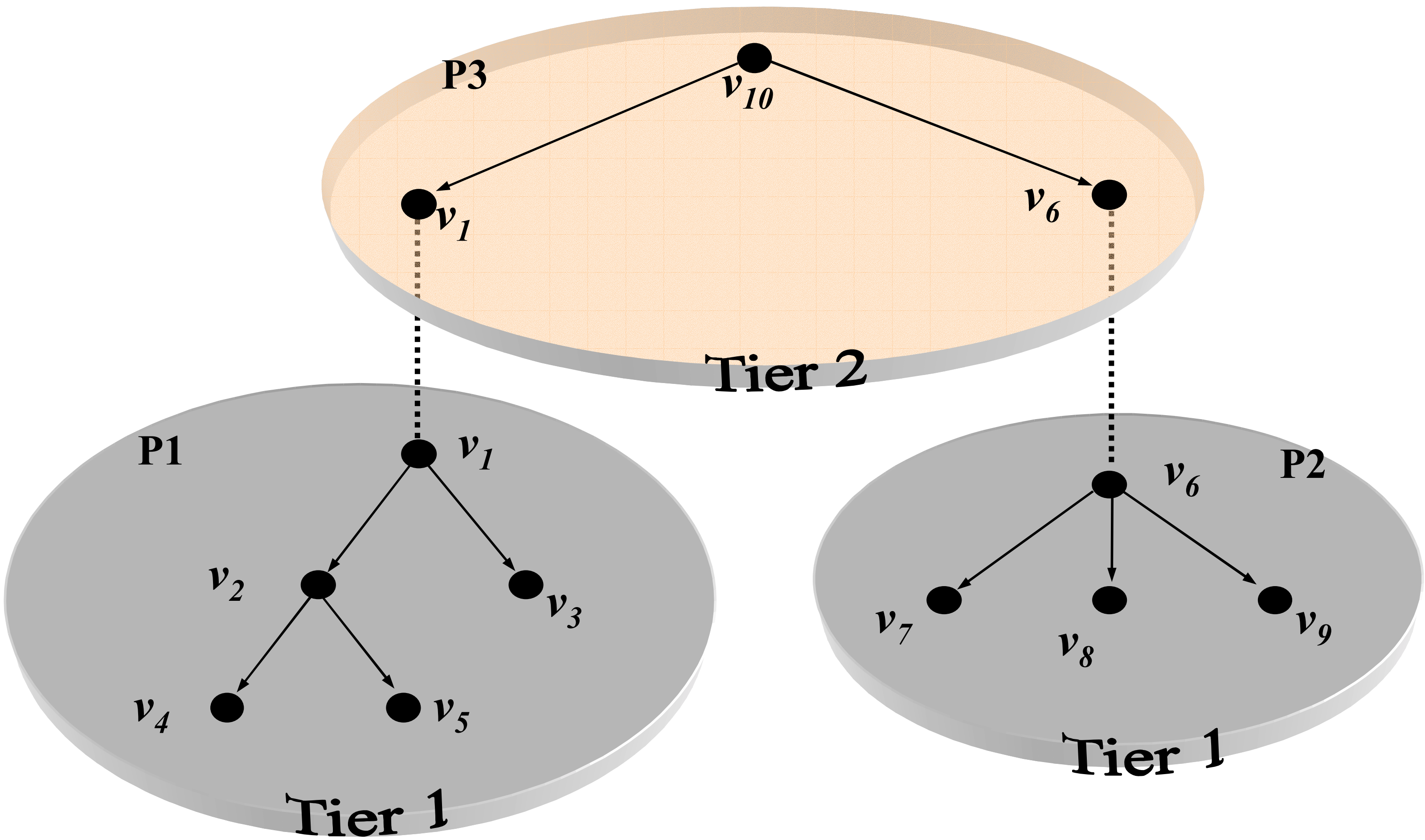
- Start with Ho-Chu 72 framework of precedence
- Adopt terminology of wireless networking community
- Consider multiple objectives via caucuses

Multiple Objectives, Caucuses and Hierarchy



Precedence diagram of flat hierarchy

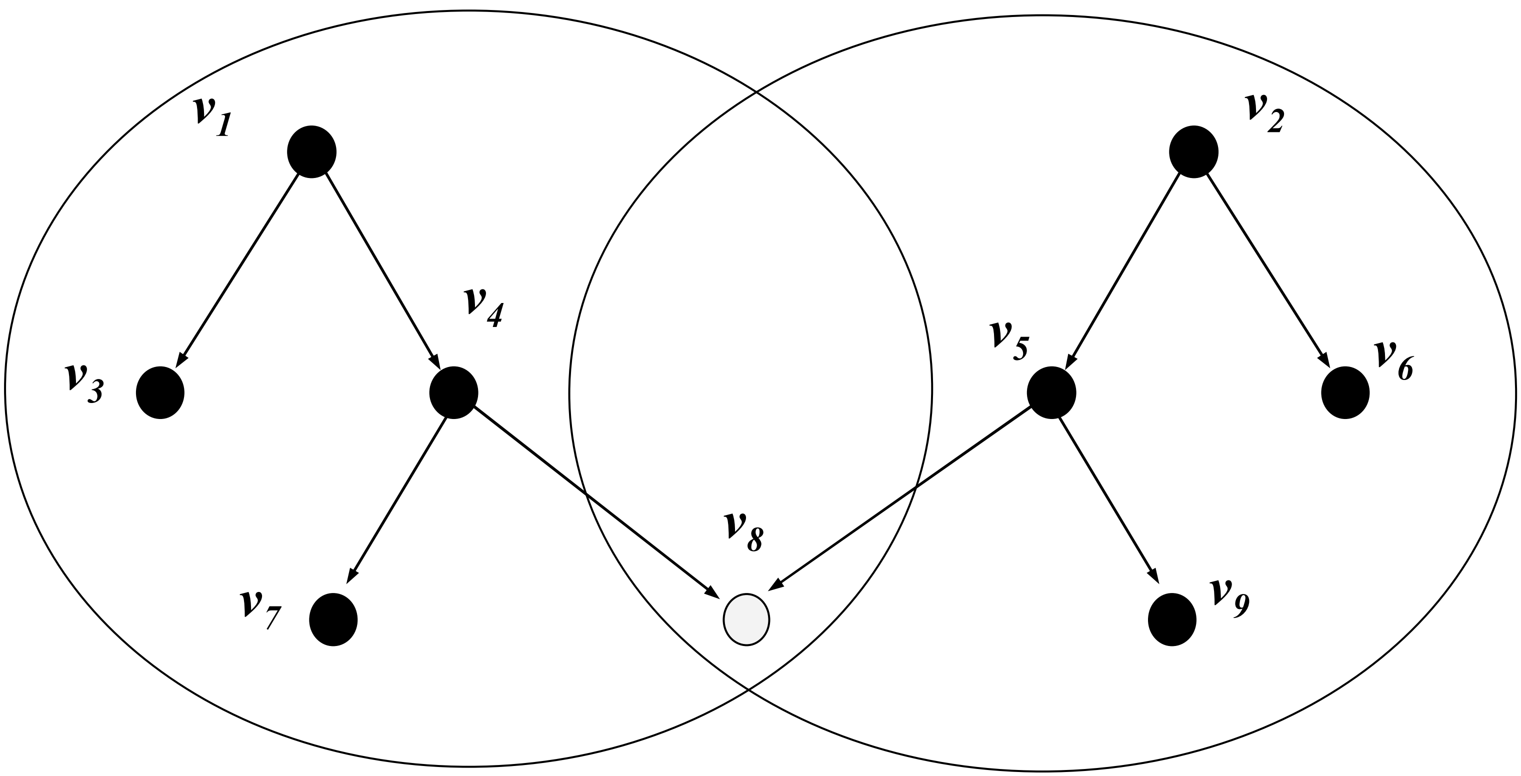
Rank Ordered Objectives



Precedence diagram of 2 tier hierarchy

Multiple Objectives, Caucuses and Hierarchy

- Issues with admissible solutions, esp. with conflict nodes at bottom of hierarchy
  - Options are pared down by predecessors.



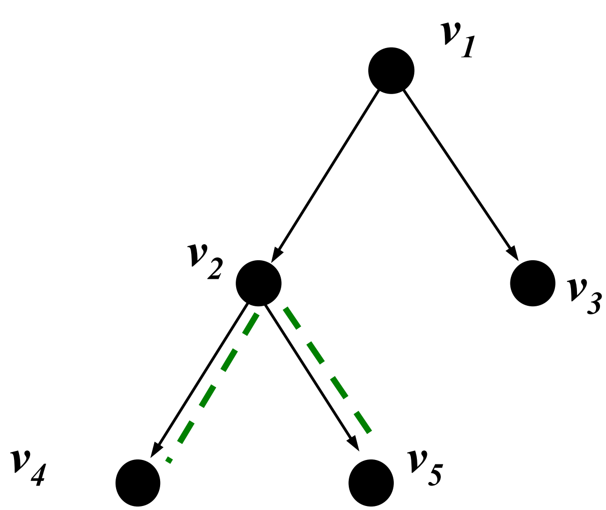
Flat hierarchy: 2 bosses example

Supervision (Heteronomous modeling)

- **Provides an alternative to precedence for influencing subordinates.**

- **Approach:**
  - **Supervisor provides subordinates with artificial objectives, constraints, or parameters (possibly functions of time) to overcome lack of information at subordinate level.**
  - **Parametric approach can be shown to be equivalent to a hybrid dynamical system**
    - **All results of hybrid systems theory then apply to cooperative supervised systems.**

Supervision (Heteronomous modeling)



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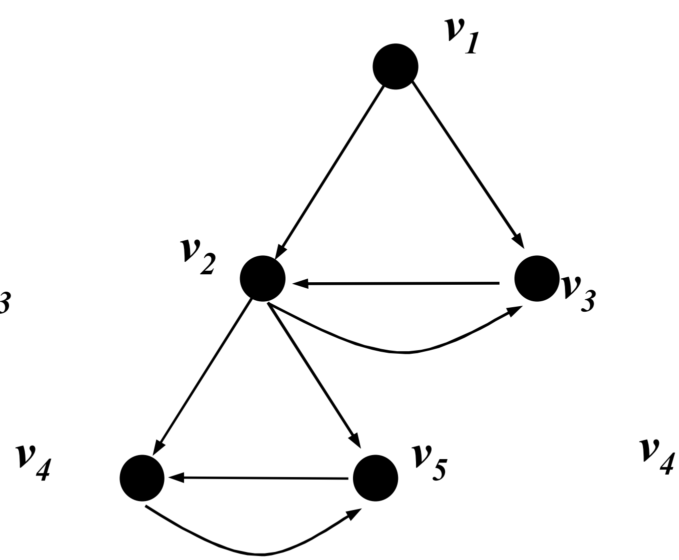

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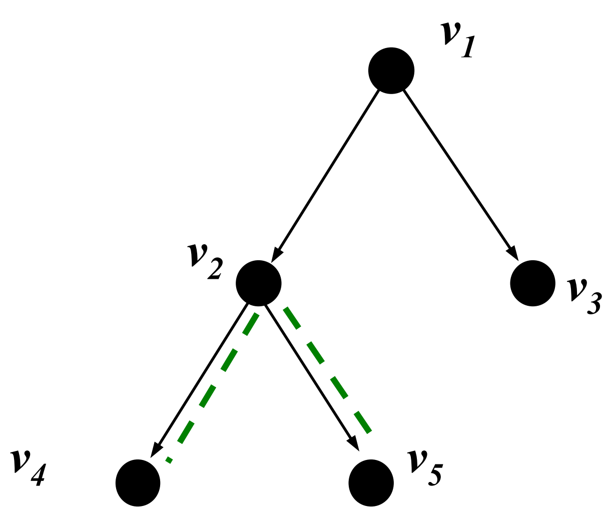

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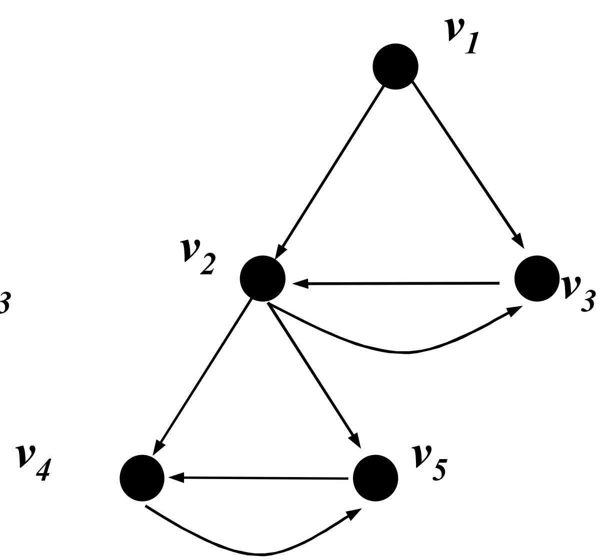

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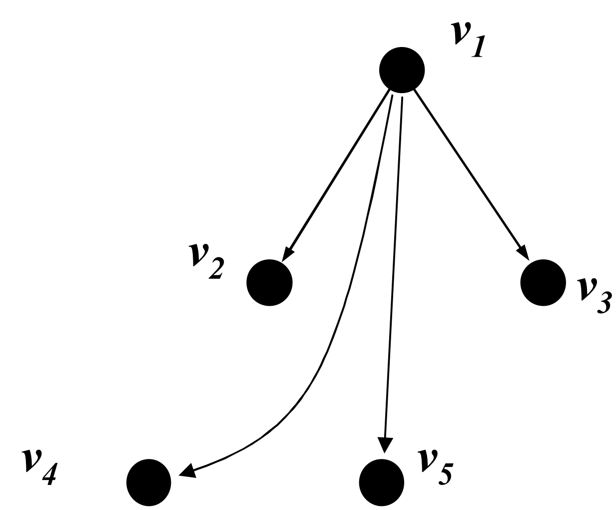

Supervision Structure

$$\dot{x}(t) = f\left(x(t), q(t)\right)$$
$$q(t) = \upsilon\left(x(t), q(t^{-})\right)$$
$$\text{where } q(t) \in Q = \{0,1,2,\ldots f\}$$

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- How are information and authority related?
- What information should be sent and to whom? → **Precedence, supervision**
- When are informationally decentralized, rule-based "emergent" approaches sufficient or optimal?
- When are cooperative systems stable? → **Hybrid systems theory**
- How do we manage uncertainty?
- How do you test cooperative systems (from simulation building up to hardware)? What are the metrics for cooperative system performance?