



CDGO 2007: 2nd International Conference on Complementarity, Duality and Global Optimization in Science and Engineering

February 28–March 2, 2007

INDUSTRIAL AND SYSTEMS ENGINEERING DEPARTMENT, UNIVERSITY OF FLORIDA



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PROGRAM

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SCHEDULE AT A GLANCE

Wednesday, February 28, 2007

0730–0815	Registration, Conference Registration Desk
0815–0830	Welcome Session
0830–0930	Plenary Speaker Christodoulos A. Floudas (Princeton)
0930–1030	Plenary Speaker Tyrrel Rockafellar (University of Washington)
1030–1045	Coffee Break
1045–1215	Sessions (Reitz Union 282, 284)
1215–1330	Lunch
1330–1420	Tutorial: Jean-Paul Penot (University of Pau, France)
1420–1435	Coffee Break
1435–1635	Sessions (Reitz Union 282, 284)
1800	Conference Reception & Dinner, Paramount Hotel

Thursday, March 1, 2007

0800–0830	Registration, Conference Registration Desk
0830–0930	Plenary Speaker Ding-Zhu Du (University of Texas)
0930–1030	Plenary Speaker David Y. Gao (Virginia Tech)
1030–1045	Coffee Break
1045–1215	Sessions (Reitz Union 282, 284)
1215–1330	Lunch
1330–1420	Tutorial: Hoang Tuy (Institute of Mathematics, Vietnam)
1420–1435	Coffee Break
1435–1635	Sessions (Reitz Union 282, 284)
1800	Conference Reception & Dinner, Paramount Hotel

Friday, March 2, 2007

0830–0930	Plenary Speaker George Isac (Royal Military College of Canada)
0930–1030	Plenary Speaker Lei Xu (Chinese University of Hong Kong)
1030–1045	Coffee Break
1045–1215	Sessions (Reitz Union 282, 284)
1215–1330	Lunch
1330–1420	Tutorial: Michael Ferris (University of Wisconsin at Madison)
1420–1435	Coffee Break
1435–1605	Session (Reitz Union 282)

Wednesday, February 28, 2007

- 0730–0815 Registration, Conference Registration Desk
- 0815–0830 **Welcome Session** (282 Lecture Hall)
Pramod P. Khargonekar, Dean of the College of Engineering, University of Florida
Panos M. Pardalos, Department of ISE, University of Florida
- Plenary Session** Chair: Panos M. Pardalos (University of Florida) (282 Lecture Hall)
- 0830–0930 **Plenary Speaker** Christodoulos A. Floudas (Princeton)
Deterministic Global Optimization: Advances and Challenges
- 0930–1030 **Plenary Speaker** Tyrrel Rockafellar (University of Washington)
Duality in Extended Linear and Nonlinear Programming
- 1030–1045 **Coffee Break**
- Session W.A1** Chair: Altannar Chinchuluun (University of Florida) (282 Lecture Hall)
- 1045–1215 Michel Thera (University of Limoges, France)
Continuation Theorems and Periodic Solutions for Evolution Variational Inequalities
- Pando Georgiev (University of Cincinnati)
Parameterized Variational Inequalities and Complementarity Problems
- Dumitru Motreanu (University of Perpignan, France)
Minimization and Variational Methods in the Study of Multiple Solutions
- Session W.B1** Chair: Sergiy Butenko (University of Texas A&M) (Room 284)
- 1045–1215 Jiawang Nie (University of Minnesota)
Semidefinite Representation of k -Ellipse
- Yubo Yuan (Virginia Tech)
Perfect Duality Theory and Optimal Solutions to a Class of Global Optimization Problems with Inequality Constraints
- Chia-Hung Chen (National Central University, Taiwan)
Lagrangian based Algorithms for Coordinated Fleet Routing and Flight Scheduling Problems
- 1215–1330 **Lunch**

Wednesday, February 28, 2007

Tutorial Session

Chair: David Y. Gao (Virginia Tech) (282 Lecture Hall)

1330–1420

Jean-Paul Penot (University of Pau, France)
Some Questions and Thoughts about Duality

1420–1435

Coffee Break

Session W.A2

Co-chairs: Mung Chiang (Princeton University) and Steven H. Low (CalTech) (282 Lecture Hall)

1435–1635

Sriram Vishwanath (University of Texas at Austin)
Converses for Certain Channel Capacity Problems using Duality

Kostas Bimpikis (MIT)
Price and Capacity Competition

Lun Li (CalTech)
Understanding Complex Networks: Principles and Models

Mung Chiang (Princeton)
Optimization Beyond Optimality: New Trends in Networking Applications

Session W.B2

Chair: Altannar Chinchuluun (University of Florida) (Room 284)

1435–1635

Chun-Ying Chen (National Central University, Taiwan)
Shipment Scheduling and Container Routing for Container Liners

Yu-Lin Shih (National Central University, Taiwan)
An Arc-Based Analogous Particle Swarm Optimization Algorithm for Minimum Cost Network Flow Problems with Concave Costs

Sabah E. Karam (Morgan State University)
A Category-Theoretic Approach to Duality

Jenna Baird (Penn State)
An Assessment of Fifty Standard Unconstrained Global Optimization Functions

1800

Conference Reception & Dinner, Paramount Hotel

Thursday, March 1, 2007

- 0800–0830 Registration, Conference Registration Desk
- Plenary Session** Chair: Asuman Ozdaglar (MIT) (282 Lecture Hall)
- 0830–0930 **Plenary Speaker** Ding-Zhu Du (University of Texas at Dallas)
Iterated 1-Steiner Trees
- 0930–1030 **Plenary Speaker** David Y. Gao (Virginia Tech)
Canonical Duality Theory and its Role in Global Optimization
- 1030–1045 **Coffee Break**
- Session T.A1** Chair: Altannar Chinchuluun (University of Florida) (282 Lecture Hall)
- 1045–1215 Mauricio Resende (AT&T Labs Research)
GRASP for Continuous Global Optimization
- Enkhbat Rentsen (National University of Mongolia, Mongolia)
Parametric Multiobjective Optimization
- Dexuan Xie (University of Wisconsin at Milwaukee)
An Adaptive Truncated Newton Method and Its Application in Computational Biochemistry
- Session T.B1** Chair: Oleg Shylo (University of Florida) (Room 284)
- 1045–1215 Nick Sahinidis (University of Illinois at Urbana-Champaign)
A Review of Optimization Techniques for Phase Retrieval based on Single-Crystal X-ray Diffraction Data
- Vikram Ganesan (General Dynamics Land Systems)
A Prototype Tool for Multidisciplinary Design Optimization of Combat Systems
- Harrison M. Kim (University of Illinois at Urbana-Champaign)
Lagrangian Coordination for Enhancing the Convergence of Multilevel Multidisciplinary Design Optimization
- 1215–1330 **Lunch**

Thursday, March 1, 2007

- Tutorial Session** Chair: Panos M. Pardalos (University of Florida) (282 Lecture Hall)
- 1330–1420 Hoang Tuy (Institute of Mathematics, Vietnam)
Robust Global Optimization
- 1420–1435 **Coffee Break**
- Session T.A2** Co-chairs: Angelia Nedich (University of Illinois at Urbana-Champaign) and Asuman Ozdaglar (MIT) (282 Lecture Hall)
- 14.35–1635 Angelia Nedich (University of Illinois at Urbana-Champaign)
Error Estimates of Approximate Primal Solutions in Dual Sub-gradient Methods
- Tamás Terlaky (McMaster University, Canada)
Central Path & Edge Path: Curvature & Diameter
- Uday Shanbhag (University of Illinois at Urbana-Champaign)
On the Solution of Nash Games under Uncertainty
- Asuman Ozdaglar (MIT)
A Geometric Framework for Nonconvex Optimization Duality using Augmented Lagrangian Functions
- Session T.B2** Chair: Zhaosheng Feng (University of Texas-Pan American) (Room 284)
- 14.35–1635 Yubo Yuan (Virginia Tech)
Perfect Duality Theory and Optimal Solutions to Non-convex Quadratic Minimization Problems with Quadratic Constraints
- Fabio Botelho (Virginia Tech)
Existence, Duality and Numerical Results for a Non-Linear Beam Model
- Zhaosheng Feng (University of Texas-Pan American)
Lie Symmetry Method and Duality Theory for A Reaction-Diffusion Equation
- Lianshuan Shi (Tianjin University of Technology and Education, China)
An Algorithm for Layout Optimization of Structures with Discrete Variables
- 1800 **Conference Reception & Dinner, Paramount Hotel**

Friday, March 2, 2007

Plenary Session

Chair: David Y. Gao (Virginia Tech) (282 Lecture Hall)

0830–0930

Plenary Speaker George Isac (Royal Military College of Canada, Canada)
About the Solvability of Complementarity Problems and Variational Inequalities Defined by Integral Operators

0930–1030

Plenary Speaker Lei Xu (Chinese University of Hong Kong, Hong Kong)
Bayesian Ying-Yang Learning: A Statistical Theory for Machine Learning with Model Selection

1030–1045

Coffee Break

Session F.A1

Chair: Oleg Shylo (University of Florida) (282 Lecture Hall)

1045–1215

Sergiy Butenko (Texas A&M University)
On a Fractional Continuous Formulation for Independence Number of a Graph

William Hager (University of Florida)
Quadratic Programming Techniques for Graph Partitioning

J. Cole Smith (University of Florida)
Optimal Mixed-Integer Programming and Heuristic Methods for a Bilevel Stackelberg Product Introduction Game

Session F.B1

Chair: Jun Zhang (University of Michigan) (Room 284)

1045–1215

Samir Adly (University of Limoges, France)
An Application of the Approach of Moreau-Panagiotopoulos in Electronics

Matsuzoe Hiroshi (Nagoya Institute of Technology, Japan)
Correspondences between Affine Differential Geometry and Information Geometry

Jun Zhang (University of Michigan)
Legendre-Fenchel Transform and Biorthogonal Coordinates of Differentiable Manifolds

1215–1330

Lunch

Friday, March 2, 2007

Tutorial Session

Chair: Panos M. Pardalos (University of Florida)

1330–1420

Michael Ferris (University of Wisconsin at Madison)

Complementarity Problems and Applications

1420–1435

Coffee Break

Session F.A2

Chair: Altannar Chinchuluun (University of Florida) (282 Lecture Hall)

14.35–1535

Xiaoqi Yang (The Hong Kong Polytechnic University, Hong Kong)

Second-Order Global Optimality Conditions for Optimization Problems

Oleg Shylo (University of Florida)

Global Equilibrium Search Applied to the Unconstrained Binary Quadratic Optimization Problem and the Weighted MAX-SAT Problem

Deterministic Global Optimization: Advances and Challenges

CHRISTODOULOS A. FLOUDAS

Department of Chemical Engineering, Princeton University

In this presentation, we will provide an overview of the research progress in global optimization. The focus will be on important contributions during the last five years, and will provide a perspective for future research opportunities. The overview will cover the areas of (a) twice continuously differentiable constrained nonlinear optimization, (b) mixed-integer nonlinear optimization, and (c) optimization with differential-algebraic models. Subsequently, we will present our recent fundamental advances in (i) convex envelope results for multi-linear functions, (ii) a piecewise quadratic convex underestimator for twice continuously differentiable functions, (iii) the generalized alpha-BB framework, and (iv) our recently improved convex underestimation techniques for univariate and multivariate functions. Computational studies will illustrate the potential of these advances.

Duality in Extended Linear and Non-Linear Programming

TYRRELL ROCKAFELLAR

Department of Applied Mathematics, University of Washington

In a view that is widely held, duality enters optimization through the introduction of Lagrange multipliers, which are tied to the imposition of equality or inequality constraints. In other words, duality in the first instance is concerned with dualization of constraints. Then further, under certain assumptions of convexity, the multipliers can be characterized as solving a dual problem. Even without convexity, the constraint dualization leads to bounds on the optimal value in the primal problem. This view, however, is unnecessarily narrow. In fact, the idea that an optimization problem should be modeled with exact constraints is too narrow. From a broader perspective, the ingredients to optimization modeling are typically some simple, structural constraints on the decision variables, expressing their nonnegativity and some definitional relationships, and then also a number of functions of these variables which all are of importance to keep high, or as the case may be, low. Faced with this, people often choose one of the functions for the objective and place constraints on the others. But there are many other approaches to modeling, which may be better, depending on the circumstances. These can include combined penalty expressions, for instance. The interesting fact is that duality can fully be developed in this “extended” setting with all the advantages that one would expect. Much greater flexibility in handling applications is thereby achieved. Once a problem is set up in the broader framework, there are easy tricks for solving it anyway by the software already available for problems in the traditional framework.

Continuation Theorems and Periodic Solutions for Evolution Variational Inequalities

MICHEL THÉRA

Department of Mathematics, University of Limoges, France

This lecture is based on a recent joint work with S. Adly and D. Goeleven [1]. In the first part, we will discuss existence results for a variational inequality of the type:

$$\text{VI}(\Lambda, \varphi) \left\{ \begin{array}{l} \text{Find } \bar{x} \in \mathbb{R}^n \text{ such that} \\ \langle \Lambda(\bar{x}), v - \bar{x} \rangle + \varphi(v) - \varphi(\bar{x}) \geq 0, \quad \forall v \in \mathbb{R}^n, \end{array} \right.$$

where $\Lambda : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous map, $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex function and $\langle \cdot, \cdot \rangle$ denotes the euclidean scalar product in \mathbb{R}^n . Then we will focus on a related class of first order evolution variational inequalities by means of a continuation method. In the cases of ordinary differential equations (ODE), M.A. Krasnosel'skii [4] and H. Amann [2], developed a continuation method to compute the Brouwer topological degree associated to some gradient mapping (called the method of guiding function). This approach was useful for the study of the existence of periodic solutions for ODE's. Roughly speaking, if on some ball of \mathbb{R}^n the Brouwer topological degree of the Poincaré translation operator associated to the ODE is different from zero, the problem has at least one periodic solution (for more details, references and possible extensions to the Leray-Schauder degree, we refer to the expository article of J. Mawhin [5]). With the emergence of many engineering disciplines and due to the lack of smoothness in many applications, it is not surprising that these classical mathematical tools require a natural extension to the class of unilateral dynamical systems.

The study of periodic solutions for evolution variational inequalities is also important. Krasnosel'skii's original approach for ODE's, has known some extensions in order to obtain continuation methods for differential inclusions (see the article of L. Gorniewicz [3] for more details and references).

In this talk, we are concerned with the existence of a T -periodic solution $u \in C^0([0, T]; \mathbb{R}^n)$ such that:

$$\begin{aligned} \frac{du}{dt} &\in L^\infty(0, T; \mathbb{R}^n); \\ u &\text{ is right-differentiable on } [0, T); \\ u(0) &= u(T); \\ \langle \frac{du}{dt}(t) + F(u(t)) - f(t), v - u(t) \rangle + \varphi(v) - \varphi(u(t)) &\geq 0, \quad \forall v \in \mathbb{R}^n, \text{ a.e. } t \in [0, T]. \end{aligned} \quad (1)$$

here, $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous map, $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex function, $f \in C^0([0, +\infty[; \mathbb{R}^n)$ is such that: $\frac{df}{dt} \in L^1_{loc}(0, +\infty; \mathbb{R}^n)$ and $T > 0$ is a prescribed period.

We prove that if F and φ satisfy some growth condition, then problem (1) has at least one periodic solution.

If we have enough time we will use this approach to obtain the existence of a T -periodic solution of a second order dynamical system of the form:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) - F(t) \in -H_1 \partial\Phi(H_1^T \dot{q}(t)), \quad (2)$$

where $q \in \mathbb{R}^m$ is the vector of generalized coordinate, $\Phi : \mathbb{R}^l \rightarrow \mathbb{R}$ is a convex function, $M \in \mathbb{R}^{m \times m}$ is a symmetric and positive definite matrix, $C \in \mathbb{R}^{m \times m}$ and $K \in \mathbb{R}^{m \times m}$ are

given matrices and $H_1 \in \mathbb{R}^{m \times l}$ is a given matrix whose coefficients are determined by the directions of friction forces. The function $F \in C^0([0, +\infty); \mathbb{R}^m)$ is such that $\frac{dF}{dt} \in L^1_{loc}([0, +\infty); \mathbb{R}^m)$. The term $H_1 \partial \Phi(H_1^T \cdot)$ is used to model the convex unilateral contact induced by friction forces.

References

- [1] S. Adly, D. Goeleven, M. Théra, *A Continuation Method for a Class of periodic evolution Variational Inequalities*, preprint 2006.
- [2] H. Amann, *A note on degree theory for gradient mappings*, Proc. Amer. Math. Soc. **85** (1982), pp 591-595.
- [3] L. Gorniewicz, *Topological Approach to Differential Inclusions*, in Topological Methods in Differential Equations and Inclusions (ed. A. Granas and M. Frigon), NATO ASI Series, Mathematical and Physical Sciences, Vol. 472 (1994), Kluwer Academic Publishers.
- [4] M.A. Krasnel'skii, *The Operator of Translation along the Trajectories of Differential Equations*, Nauka, Moscow (1966) (Russian); english translation: American Math. Soc., Translations of math. Monographs, vol. 19, Providence, 1968.
- [5] J. Mawhin, *Continuation Theorems and Periodic Solutions of Ordinary Differential Equations*, in Topological Methods in Differential Equations and Inclusions (ed. A. Granas and M. Frigon), NATO ASI Series, Mathematical and Physical Sciences, Vol. 472 (1994), Kluwer Academic Publishers.

Continuation Theorems and Periodic Solutions for Evolution Variational Inequalities

PANDO GEORGIEV

ECECS Department, University of Cincinnati

We consider variational inequalities defined by monotone operators depending on a parameter and describe conditions under which a perturbed variational inequality has a solution depending in a continuous way on the parameter. Applications are given to classes of complementarity problems and minimax problems.

Minimization and Variational Methods in the Study of Multiple Solutions

DUMITRU MOTREANU

Department of Mathematics, University of Perpignan, France

Consider the nonlinear eigenvalue problem with Dirichlet boundary condition: Find $u \in W_0^{1,p}(\Omega) \setminus \{0\}$ and $\lambda < 0$ such that,

$$-\Delta_p u = f(x, u, \lambda) \text{ in } \Omega, u = 0 \text{ on } \partial\Omega \quad (3)$$

Where $\Omega \subset \mathbb{R}^N$ is a bounded domain of class \mathbb{C}^2 , Δ_p is the p -Laplacian on

$$W_0^{1,p}(\Omega) (1 < p < +\infty)$$

and

$$f : \Omega \times \mathbb{R} \times (0, \bar{\lambda}) \rightarrow \mathbb{R} (\bar{\lambda} > 0)$$

is a given function satisfying the growth condition (H): there exist constants $c > 0, r > p - 1$, and functions $a(\cdot, \lambda) \in L^\infty(\Omega)_+ (\lambda \in (0, \bar{\lambda}))$ such that $\|a(\cdot, \lambda)\|_\infty \rightarrow 0$ as $\lambda \downarrow 0$ and $|f(x, s, \lambda)| \leq a(x, \lambda) + c|s|^r$ p.p. $x \in \Omega$ et pour tout $(s, \lambda) \in \mathbb{R} \times (0, \bar{\lambda})$.

Under assumption (H) the functional associated to problem 3 is not defined, so the usual variational methods cannot be directly applied to problem 3 in our setting. Using the Morse theory, under suitable hypotheses, Jin [3] has shown (for $p = 2$) that there exist at least three nontrivial solutions of problem 3 for every λ sufficiently small, two of them being of opposite constant sign. By a technique based on minimization with truncation combined with variational methods, Motreanu-Motreanu-Papageorgiou [4] have shown that (for each $p > 1$) the third nontrivial solution can be found in the order interval determined by the two nontrivial opposite constant sign solutions. Strengthening the hypothesis describing the behavior near 0 of $f(x, \cdot, \lambda)$, Carl-Motreanu [2] have obtained the existence of three nontrivial solutions, two being of opposite constant sign and the third one being of changing sign on Ω . The basic argument in this result is supplied by certain comparison principles (see Carl-Le-Motreanu [1]) ensuring that the nontrivial constant sign solutions can be chosen to be extremal in an appropriate sense.

References

- [1] S. Carl, V. K. Le and D. Motreanu, Nonsmooth variational problems and their inequalities. Comparison principles and applications, Springer, New York, 2007.
- [2] S. Carl and D. Motreanu, Constant-sign and sign-changing solutions of a nonlinear eigenvalue problem involving the p -Laplacian, Differential Integral Equations, in print.
- [3] Z. Jin, Multiple solutions for a class of semilinear elliptic equations, Proc. Amer. Math. Soc. 125 (1997), 36593667.
- [4] D. Motreanu, V. V. Motreanu and N. S. Papageorgiou, Multiple nontrivial solutions for nonlinear eigenvalue problems, Proc. Amer. Math. Soc., in print.

Semidefinite Representation of k-Ellipse

JIAWANG NIE

IMA, University of Minnesota

K-ellipse is the set of all points whose summation of distances to k fixed points is a constant. This talk will give a semidefinite representation of the polynomial equation and the region bounded by k -ellipse. A linear matrix inequality and degree formulae will be given.

Perfect Duality Theory and Optimal Solutions to a Class of Global Optimization Problems with Inequality Constraints

DAVID YANG GAO AND YUBO YUAN

Department of Mathematics, Virginia Tech

This paper presents a perfect duality theory and optimal solutions to a class of global optimization problems subjected to linear inequality constraints. These problems are directly related to numericalization of a large class of semi-linear nonconvex partial differential equations in nonconvex mechanics including phase transitions, chaotic dynamics, nonlinear field theory, and superconductivity. By use of the canonical dual transformation developed recently, a canonical dual problem is formulated, which is perfectly dual to the primal problem. The global minimizer of the nonconvex forth function can be identified by the triality theory. Results show that if the global extrema of the nonconvex forth function are located on the boundary of the primal feasible space, the dual solutions should be interior points of the dual feasible set, which can be solved by deterministic methods. It turns out that perfect duality theory and optimal solutions to a Class of global optimization problems with inequality constraints are obtained. Several examples are illustrated to show the beauty of this theory.

Lagrangian based Algorithms for Coordinated Fleet Routing and Flight Scheduling Problems

CHIA-HUNG CHEN¹, SHANGYAO YAN², AND MIAWJANE CHEN³

¹*Department of Civil Engineering, National Central University, Taiwan*

²*Department of Civil Engineering, National Central University, Taiwan*

³*Department of Finance, National United University, Taiwan*

We develop a mixed-stop heuristic and a family of Lagrangian based algorithms, based on Lagrangian relaxation, a subgradient method, and four heuristics for the upper bound solution, for solving coordinated fleet routing and flight scheduling problems. Numerical tests based on real operating data from two Taiwan airlines were performed to evaluate the proposed solution algorithms. The test results indicate that these solution algorithms are a significant improvement over CPLEX. Moreover, the Lagrangian based algorithms are better than the mixed-stop heuristic; they could be useful for the allied airlines to solve coordinated fleet routing and flight scheduling problems.

Some Questions and Thoughts about Duality

JEAN-PAUL PENOT

Department of Applied Mathematics, University of Pau, France

Duality is an important tool in optimization theory and practice. But what is precisely a duality? What means are available to obtain duality results? Is there a sensible notion of subdifferential attached with a duality? What are the main links with variational analysis?

We endeavour to give answers to such questions. We illustrate our analysis with some examples in which some relaxed forms of convexity play some role.

References

- [1] J.-P. Penot, What is quasiconvex analysis?, *Optimization* 47 (2000), 35-110.
- [2] J.-P. Penot, Duality for anticonvex programs, *J. of Global Optim.* 19 (2001), 163-182.
- [3] J.-P. Penot, Rotundity, smoothness and duality, *Control and Cybernetics* 32 (4) (2003), 711-733.
- [4] J.-P. Penot, Unilateral analysis and duality, in *GERAD, Essays and Surveys in Global Optimization*, G. Savard et al eds., Springer, New York (2005), 1-37.
- [5] J.-P. Penot, The Legendre transform of correspondences, *Pacific J. Optim.* 1 (1)(2005), 161-177.
- [6] J.-P. Penot, Ekeland duality as a paradigm, *J. Global Optimization*, to appear.
- [7] J.-P. Penot and C. Zalinescu, Continuity of the Legendre-Fenchel transform for some variational convergences, *Optimization* 53 (5-6) (2004), 549-562.

Converses for Certain Channel Capacity Problems using Duality

SRIRAM VISHWANATH¹ AND SHLOMO (SHITZ) SHAMAI²

¹ *Department of Electrical and Computer Engineering, University of Texas at Austin*

² *Department of Electrical Engineering, Technion, Israel*

In this talk, we discuss the use of Lagrangian duality to establish a converse for the information-theoretic capacity (regions) of certain channels whose capacities are otherwise difficult to characterize. An example of such a channel is the channel with state where a noisy version of the state information is known at the transmitter. Other example channels will be given in the talk as well. We will also characterize coding schemes that achieve the outer bounds we derive using duality theory.

Price and Capacity Competition

ASUMAN OZDAGLAR, DARON ACEMOGLU AND KOSTAS BIMPIKIS

Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology

We study the efficiency of oligopoly equilibria in a model where firms compete over capacities and prices. The motivating example is a communication network where service providers invest in capacities and then compete in prices. Our model economy corresponds to a two-stage game. First, firms (service providers) independently choose their capacity levels. Second, after the capacity levels are observed, they set prices. Given the capacities and prices, users (consumers) allocate their demands across the firms. We first establish the existence of pure strategy subgame perfect equilibria (oligopoly equilibria) and characterize the set of equilibria. These equilibria feature pure strategies along the equilibrium path, but off-the-equilibrium path they are supported by mixed strategies. We then investigate the efficiency properties of the equilibria, where "efficiency" is defined as the ratio of surplus in equilibrium relative to the first best.

We also show that an alternative game form where capacities and prices are chosen simultaneously always fails to have a pure strategy equilibrium. Our results suggest that the timing of capacity and price choices in oligopolistic environments is important both for the existence of equilibrium and for the extent of efficiency losses in equilibrium.

Understanding Complex Networks: Principles and Models

LUN LI¹, STEVEN H. LOW¹, JOHN C. DOYLE¹, AND WALTER WILLINGER²

¹ *Department of Computer Science and Electrical Engineering, California Institute of Technology*

² *AT&T Labs*

There is a large, popular, and growing literature on "scale-free" networks with the Internet along with metabolic networks representing perhaps the canonical examples. While this has in many ways reinvigorated graph theory, there is unfortunately no consistent, precise definition of scale-free graphs and few rigorous proofs of many of their claimed properties. In fact, it is easily shown that the existing theory has many inherent contradictions and that the most celebrated claims regarding the Internet and biology are verifiably false. We provided a completely different view of the Internet, by introducing an engineering approach that combines notions of optimization, control and dynamics. We introduce a structural metric that allows us to differentiate between all simple, connected graphs having an identical degree sequence, which is of particular interest when that sequence satisfies a power law relationship. We demonstrate that the proposed structural metric yields considerable insight into the claimed properties of SF graphs and provides one possible measure of the extent to which a graph is scale-free. This structural view can be related to previously studied graph properties such as the various notions of self-similarity, likelihood, betweenness and assortativity. Our approach clarifies much of the confusion surrounding the sensational qualitative claims in the current literature, and offers a rigorous and quantitative alternative, while suggesting the potential for a rich and interesting theory.

Optimization Beyond Optimality: New Trends in Networking Applications

MUNG CHIANG

Department of Electrical Engineering, Princeton University

Optimization of communication networks has witnessed an impressive growth of research activities over the last ten years. In addition to viewing networks as objects to be optimized, some of these works also view networks as optimizers themselves. In addition to “Design by Optimization”, some recent results also demonstrate the principle of “Design for Optimizability”. Beyond solving a problem for optimality, optimization theory has also provided to networking applications the followings: a modeling language for design, a reverse-engineering methodology for analysis, an analytic framework for architectural decisions, a quantitative basis for fairness and robustness, and even an indicator of flaws in engineering assumptions. Many of these new uses of optimization actually do not involve solving any problem for optimality. Drawing successful applications ranging from Internet TCP/IP control to protocol stacks of wireless ad hoc networks, this talk will survey these emerging trends that give rise to many new meanings of the phrase “optimization of networks”.

Shipment Scheduling and Container Routing for Container Liners

SHANGYAO YAN, CHUN-YING CHEN, AND SHIH-CHUN LIN

Department of Civil Engineering, National Central University, Taiwan

A good shipment scheduling and container routing plan is very important for container liners. However, most such carriers in Taiwan currently adopt a trial-and-error process for shipment scheduling and container routing practices, which is neither efficient nor effective. We employ network flow techniques to construct a multiple fleet-flow network and multiple cargo-flow network model, in which the flows of ships and cargos are formulated in the dimensions of time and space. Some side constraints between the fleet- and cargo-flow networks are set to comply with real operating requirements. The model is formulated as a multiple commodity network flow problem, characterized as NP-hard. A heuristic, based on Lagrangian relaxation, a subgradient method and heuristics for the upper bound solution, is developed to solve the model. The test results, based on data from a major Taiwan marine shipping company operation, show the good performance of the model and the solution algorithm.

An Arc-Based Analogous Particle Swarm Optimization Algorithm for Minimum Cost Network Flow Problems with Concave Costs

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In this research, we employ particle swarm optimization algorithm, coupled with the techniques of genetic algorithm, threshold accepting method and concave cost network heuristics, to develop an Arc-based global search algorithm that can efficiently solve minimum cost network flow problems with concave costs. To evaluate the proposed algorithm, several network flow problems are randomly generated. The C++ computer language is used to code all the necessary programs for the tests. The results indicate that the proposed algorithm is more effective than genetic algorithm, threshold accepting method and great deluge algorithm for solving minimum cost network flow problems with concave costs.

Duality: A Category-Theoretic approach to Duality

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Planning & Information Technology, Morgan State University

Category theory has revealed significant structural relations in mathematics, computer science, quantum mechanics, and biology. It is a theory that deals in an abstract way with mathematical structures and relationships between them. Duality concepts will be examined from the perspective of category theory. Notions of duality, taken from a variety of disciplines, will be analyzed using a category theoretic approach that reveals structure preserving processes and other natural transformations. Propositions of Duality will be derived from the category theory analysis.

An Assessment of Fifty Standard Unconstrained Global Optimization Functions

JENNA BAIRD AND MADARA OGOT

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Standard unconstrained global optimization problems are benchmark functions whose answers are known and therefore allow researchers to compare the relative efficiency of their global optimization algorithms. They are all multimodal nonlinear problems and it is generally not possible to view them and know in advance if it will be easy or difficult to find their global minimum (or maximum). In the literature, difficulty of solution is often measured along two dimensions, ability to consistently find the global minimum, and the computational effort to do so. Despite the large number of standard global optimization problems in the literature, however, few attempts have been made to classify them vis--vis, level of difficulty. As a result researchers in the field typically make assumptions (often erroneously) on a problems level of difficulty, leading to wrong parameter settings for their global optimization routines and the reporting of incorrect results. Based on earlier categorization work by Trn, Ali and Viitanen (1999), this study sought to classify fifty commonly used unconstrained global optimization problems as compiled in Ali, Khompatraporn, and Zabinsky (2005).

Using the gradient-based local optimization routine `fmincon` found in the MATLAB optimization toolbox, n local runs from random points in the design space were performed and the number of times the global minimum located noted, m . The ratio, $p^* = m/n$ was calculated as a measure of difficulty level. Starting at 5,000 runs, n was continuously increased until p^* reached a constant value. Torn et al. (1999) rationalized the use of $(1-p^*)$ as one parameter used in combination with other measures such as global minimum embeddedness or global minimum isolation, and the number of the local minima to collectively used to classify the functions into three broad categories: easy, medium and difficult. However, no indication was given as to the cut-off points between the different levels. In this study, we propose the use of $\log(p^*)$ as a more logical parameter. The full paper includes the rationale behind this decision as well as the proposed cut-off points between levels. For each of the 50 standard problems, therefore $\log(p^*)$, global minimum embeddedness or global minimum isolation, and the number of the local minima were determined, and the problem placed into each one of the broad categories.

Knowing the level of difficulty of finding the global solution to the standard functions should allow researchers developing global optimization algorithms to be able to (1) set algorithm parameters correctly during the fine-tuning phase of development, and (2) determine what type of problems their algorithms are best suited for.

Iterated 1-Steiner Trees

DING-ZHU DU

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The iterated 1-Steiner tree is a heuristic for the Steiner minimum tree. It was proposed in 1972 and has been found to have very good performance in computational experiments. However, the theoretical analysis of it was a long-standing open problem. In this talk, we give a solution for it.

Canonical Duality Theory and Its Role in Global Optimization

DAVID YANG GAO

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Canonical duality theory is a newly developed, potentially powerful methodology, which is composed mainly of a canonical dual transformation and a triality theory. The canonical dual transformation can be used to formulate perfect dual problems without duality gap, while the triality theory reveals an interesting duality pattern in general nonconvex system and plays a fundamental role in nonconvex analysis and global optimization.

In this talk, the speaker will present a comprehensive review and some new developments on the canonical duality theory and its applications in global optimization. By using the most simple but fundamentally difficult (NP-hard) quadratic programming problem, the speaker will reveal a unified structure and a splendid beauty in mathematical programming. He will show that by using the canonical dual transformation, many well-known nonconvex/nonsmooth problems in high dimensional space can be reformulated into certain smooth canonical dual problems in lower dimensional space; integer programming problems can be converted to certain continuous dual problems; a large class of constrained nonlinear optimization problems can be assembled in a unified framework. An insightful relation between the canonical dual transformation and nonlinear (or extended) Lagrange multiplier methods is presented. The triality theory can be used to identify both global and local optimizers and to develop some potentially powerful algorithms for solving many challenging problems.

Extensive applications will be illustrated by general nonconvex constrained problems in global optimization. Complete solutions to certain well-known difficult problems will be presented, including polynomial minimization, nonconvex programming with box, integer, and multi-quadratic constraints. This talk should bring some fundamentally new insights into global optimization and computational science.

GRASP for Continuous Global Optimization

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PARDALOS³

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We describe a method adapted from Feo and Resende's GRASP metaheuristic, first proposed for solving combinatorial optimization problems, to solve global optimization problems with continuous objective functions and box constraints. In computational experiments, the algorithm is shown to almost always avoid the trappings of local minima, converging to a global minimum. This method is very easy to implement, is applicable to a wide range of global optimization problem, and does not make use of derivative information, thus making it a well-suited general approach for solving global optimization problems.

Parametric Multiobjective Optimization

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Multiobjective optimization problems are widely used not only in mathematics but also in engineering and economics. Pareto optimal solutions play important role in multiobjective optimization. In this talk, the weighted sum approach for finding a Pareto optimal solutions has been presented depending on a parameter value. We show that the one parametric optimization techniques can be applied to parametric multiobjective optimization. Some numerical examples are provided.

An Adaptive Truncated Newton Method and Its Application in Computational Biochemistry

DEXUAN XIE

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This talk will introduce a new adaptive truncated-Newton method (A-TN) for solving large scale unconstrained minimization problems with dense Hessian matrices. A-TN is a variant of the truncated-Newton method in which the Hessian matrix is replaced by a sparse approximate Hessian matrix constructed adaptively according to an available computing resource. For two particularly important application problems in computational biochemistry, the chemical database problem and the biomolecular potential energy minimization problem, a good approximate Hessian matrix is constructed by using a distance cutoff strategy, resulting in an efficient and effective A-TN algorithm. A mathematical analysis shows that A-TN is convergent globally with at least a linear rate. Numerical results demonstrate that A-TN can be more efficient and more numerically stable than both the quasi-Newton BFGS method and the discrete truncated-Newton method.

A Review of Optimization Techniques for Phase Retrieval based on Single-Crystal X-ray Diffraction Data

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A vast majority of molecular structures are derived using the physics of X-ray diffraction, especially in the case of important biological macromolecules. The computation of a structure from X-ray diffraction data, however, remains a very challenging, nontrivial problem, both experimentally and computationally. A major obstacle, traditionally coined "the phase problem" in the computation of the 3D structure of a crystal, results from that fact that phase information is lost and not directly measurable in a traditional X-ray diffraction experiment. The primary focus of this paper is in cataloguing the major models and techniques currently used in phase determination from single-crystal X-ray diffraction data, along with a critical analysis in regard to optimization problems present. First, an overview of each problem is given, including the significance, chronologically, for each method in terms of pushing the boundaries of solvable structures in the past. Then, focus is turned to optimization models at the crux of each method, with a critical assessment of each problem's contemporary standing and suggestions for future work in improving on published techniques. Specifically, this paper looks at the use of the minimum superposition function for the solution of Patterson maps, the problem of heavy atom selection for MAD/SAD techniques, the general minimal principle formulation, an integer minimal principle for centrosymmetric structures, maximum determinant methods, and maximum likelihood techniques.

A Prototype Tool for Multidisciplinary Design Optimization of Combat Systems

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This work defines and investigates a Multidisciplinary Design Optimization (MDO) scheme for an integrated combat systems design environment. The model developed is low-fidelity, and employs surrogate parametric analysis modules to examine problem formulation issues prior to integration with higher fidelity modules. The problem formulation includes modules for Mobility, Human Factors, Power and Energy Management, Transportability, Mass and Dimensional Properties, Thermal Management, Survivability and Lethality. The objective function is the system effectiveness of the combat system. Constraints have been imposed to satisfy Government compliance requirements and regulations. The system is flexible to allow user-defined variables pertaining to technology, usability, economic and geographic conditions. To solve the problem, traditional nonlinear algorithms, and global optimization techniques are employed. The overall intention is to develop a first principles and statistical analysis-based platform to perform effective design synthesis on the basis of simultaneous consideration of conflicting design requirements, to perform sensitivity analysis, and to identify a global optimal design.

Lagrangian Coordination for Enhancing the Convergence of Multilevel Multidisciplinary Design Optimization

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Analytical target cascading (ATC) is a hierarchical, multilevel, multidisciplinary methodology for large-scale system design. After a large-scale system design problem is decomposed in a hierarchical manner, hierarchical decision making (i.e., cascading targets) is then modeled as individual design optimization problems at each level, possibly in multiple subproblems. The focus of this presentation is on a new Lagrangian duality-based coordination algorithm that improves the convergence of ATC to enforce consistency between subproblems at multiple levels. The proposed approach guarantees all the properties established earlier but additionally offers new significant advantages.

In the context of multidisciplinary design optimization (MDO), ATC can be viewed as a hierarchical multilevel formulation as well as a coordination methodology. In ATC, starting from the top-level targets, an optimization problem at the top level is solved to find an optimal design and subtargets for the immediate lower-level subsystems. At the next lower level, the objective is defined as the minimum deviation from the upper-level targets with respect to the current responses and linking variables. Once the lower-level problem has been solved, the adjusted responses and linking variables are returned to the upper level so that the overall responses are adjusted based on the feasibility of lower-level design.

This work follows upon the study by Lassiter et al. who placed ATC in the historical context of Lagrangian decomposition and coordination methods for large-scale systems, proposed an ATC-based decomposition and coordination approach based on Lagrangian duality theory, and established its convergence. In this presentation, a revised Lagrangian-based ATC formulation and a solution process will be proposed. In the formulation, the Lagrange multipliers can be viewed as the weights for deviations in ATC formulations. Thus the proposed coordination algorithm finds the optimal solution and the optimal weights for the deviation terms simultaneously. The enhancement allows for target cascading between levels, the use of augmented Lagrangian to improve convergence of the coordination algorithm, and for prevention from unboundedness. A guideline to set the step size for subgradient optimization when solving the Lagrangian dual problem will be also proposed.

Similar to the approach of Lassiter et al., the revised formulation guarantees that under convexity assumptions for the original design objective and constraints, the resulting Lagrangian coordination algorithm finds a unique global optimum as well as (optimal) weights for the ATC deviation terms. Two demonstration cases show that in the proposed Lagrangian coordination the weights do not necessarily need to be large and that for a convex ATC-decomposed problem there exists a unique set of weights to achieve the pre-specified tolerance.

Robust Global Optimization

HOANG TUY

Institute of Mathematics, Vietnam

A wide class of global optimization problems have the form

$$\min\{f(x)|g(x) \leq 0\} \quad (4)$$

where $g(x) = \max_{i=1,\dots,m} g_i(x)$, and $f, g_i : \mathbb{R}^n \rightarrow \mathbb{R}$. Since computing a feasible solution to (4) may be very hard, most solution methods for (4) compute, for given tolerances $\epsilon > 0, \eta > 0$, an (ϵ, η) - approximate optimal solution, i.e. a point \bar{x} such that :

1. \bar{x} is an ϵ -feasible solution, in the sense that $g(\bar{x}) \leq \epsilon$;
2. $f(\bar{x}) \leq f(x) + \epsilon$ for all ϵ -feasible solutions x .

However this concept of approximation may not be quite adequate, since such an (ϵ, η) - approximate optimal solution may correspond to an objective function value far from the true optimal value, while being infeasible. We introduce a concept of essential ϵ - optimal solution, which corresponds to a more appropriate approximation of the optimal global solution, while being stable under small perturbations of the constraints. A general robust solution approach is proposed which can be applied whenever $f, g_i \in \mathcal{C}$, where \mathcal{C} is a family of functions with the following property:

1. If $f \in \mathcal{C}, \alpha \in \mathbb{R}_+$ then $\alpha f \in \mathcal{C}$; If $g_1, g_2 \in \mathcal{C}$ then $\max g_1(x), g_2(x) \in \mathcal{C}$;
2. Any set of the form $D = \{x \in \mathbb{R}^n | h(x) \leq 0\}$ with $\text{int } D \neq \phi$; satisfies $\text{cl}(\text{int } D) = \text{cl}(D)$.

This property ensures that the family $\mathcal{F}(\mathcal{C}) := \mathcal{C} - \mathcal{C}$ is a vector lattice and if every function f, g_i involved in a problem (4) are functions of the family $\mathcal{F}(\mathcal{C})$ then computing an essential ϵ - optimal solution reduces to solving a sequence of robust problems of the form $\min\{f(x)|g(x) \leq 0\}$ with $h \in \mathcal{C}, g \in \mathcal{F}(\mathcal{C})$. Examples of \mathcal{C} :

1. family of convex functions
2. family of increasing functions

Numerical examples are discussed to illustrate the approach.

Error Estimates of Approximate Primal Solutions in Dual Subgradient Methods

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This talk is focused on a general constrained optimization problem with convex objective function and convex inequality constraints that are not necessarily differentiable. We consider the dual problem arising from Lagrangian relaxation of the inequality constraints. We apply subgradient methods to the dual problem and we generate approximate primal solutions as a byproduct of the subgradient method and an averaging scheme.

We propose and analyze two such approaches assuming the existence of a Slater vector for the original constrained problem. For each of these approaches, we provide error estimates associated with the resulting approximate primal solutions. Specifically, we provide upper and lower bound estimates for the approximations of the primal optimal value. We also provide estimates for the violation of the primal inequality constraints. We conclude the discussion by comparing the two approaches and by discussing their potential advantages.

Central Path & Edge Path: Curvature & Diameter

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It was shown recently that the central path can be bent along the simplex path of Klee-Minty cubes. This lead to tightening the iteration complexity bound of central path following interior point methods. Further, intriguing analogs between edge-paths and central paths arise. We conjecture that the order of the largest total curvature of the central path is the number of inequalities, and that the average diameter of a bounded cell of an arrangement is less than the dimension. We substantiate these conjectures and prove a continuous analog of the d -step conjecture.

On the Solution of Nash Games under Uncertainty

UDAY SHANBHAG

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We consider a Nash equilibrium problem in which agents solve two-stage stochastic convex optimization problems. Under an assumption of a discrete measure, the resulting equilibrium conditions may be posed as a finite-dimensional complementarity problem, albeit an arbitrarily large one. We present a decomposition-based approach for solving such a class of complementarity problems. Convergence theory is provided along with some preliminary computational experience.

A Geometric Framework for Nonconvex Optimization Duality using Augmented Lagrangian Functions

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We provide a unifying geometric framework for the analysis of general classes of duality schemes and penalty methods for nonconvex constrained optimization problems. We present separation results for nonconvex sets via general concave surfaces. We use these separation results to provide necessary and sufficient conditions for establishing strong duality between geometric primal and dual problems. Using the primal function of a constrained optimization problem, we apply our results both in the analysis of duality schemes constructed using augmented Lagrangian functions, and in establishing necessary and sufficient conditions for the convergence of penalty methods.

Perfect Duality Theory and Optimal Solutions to Non-convex Quadratic Minimization Problems with Quadratic Constraints

YUBO YUAN AND DAVID YANG GAO

Department of Mathematics, Virginia Tech

This paper presents a perfect duality theory and optimal solutions to non-convex quadratic minimization problems with quadratic constraints. It is directly related to subproblem of trust region algorithm, economic equilibrium problem, combinatorial optimization, numericalization of a large class of nonconvex partial differential equations, and general nonlinear programming are all sources of quadratic optimization. By using of the canonical dual transformation developed recently, a canonical dual problem is formulated, which is perfectly dual to the primal problem. The global minimizer can be identified by the triality theory. Results show that if the global extrema are located on the boundary of the primal feasible space, the dual solutions should be interior points of the dual feasible set, which can be solved by deterministic methods. It turns out that perfect duality theory and optimal solutions to a class of global optimization problems with quadratical constraint are obtained. Several examples are illustrated to show the beauty of this theory.

Existence, Duality and Numerical Results for a Non-Linear Beam Model

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This article develops dual variational formulations for a non-linear beam model. The concerned model was developed by Gao [6] and, in its boundary value form, consists of a cubic semi-linear differential equation of fourth order and associated boundary conditions. It is important to emphasize that the correspondent primal variational formulation is non-convex, of double-well type. Two duality principles are presented. The first one is obtained through an application of Gao's Canonical Duality Method, whereas the second duality principle is obtained through a combination of classical results of convex analysis, following Rockafellars and Tolands notions of duality. Also we present existence and numerical results for post-buckling analysis for the mentioned model.

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Lie Symmetry Method and Duality Theory for the Complex Ginzburg-Landau Equation

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In this talk, we apply the Lie symmetry method and duality theory to the study of a reaction-diffusion equation. This equation can be regarded as a generalization of the Fisher equation, which is used as a density-dependent diffusion model, in the one-dimensional situation, for studying insect and animal dispersal with growth dynamics, and as a genetic model arising from the classical theory of population genetics and combustion. The traveling wave phenomena are described and a class traveling wave solution is derived in explicit functional forms. Numerical simulations for approximate solutions are also illustrated.

An Algorithm For Layout Optimization of Structures with discrete variables

LIANSHUAN SHI

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In this talk, a mathematical model for Layout optimization of structure with discrete variables is presented. The optimization procedure is composed of two kinds of sub-procedures of optimization: the topological optimization and the shape optimization. The two kinds of optimization procedures are applied in turn until convergence appears. After the dimension of the structure is reduced, the delimiting combinatorial algorithm is used to search for the better objective value. Several realistic examples will also be illustrated.

About the Solvability of Complementarity Problems and Variational Inequalities Defined by Integral Operators

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In many practical problems considered in sciences and engineering, generally related to equilibrium or to unilateral boundary condition, we have to study solvability of variational inequalities or of complementarity problems defined by integral operators. Also, we have variational inequalities or complementarity problems with eigenvalues. Related to this reality we will present in this talk some topological methods applicable to the study of solvability of complementarity problems and of variational inequalities in Hilbert Spaces. The main chapters of our talk are:

1. Variational inequalities defined by integral operators, exceptional families of elements, the Harker-Pang type condition and solvability.
2. Scalar asymptotic derivability and a fixed point theorem applicable to complementarity problems defined by integral operators.
3. Scalar compactness, condition and variational inequalities.
4. Quasi-bounded operators and complementarity problems defined by integral operators and depending of parameters (i.e., with eigenvalues).
5. Comments

Some open subjects will be also presented as stimulus for new developments related to the interaction between nonlinear analysis and complementarity problems defined by integral operators.

A Statistical Theory for Machine Learning with Model Selection

LEI XU

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Conventionally, the tasks of learning include a parameter learning part for determining unknown parameters and a model selection part for selecting an appropriate scale for a model that accommodates these parameters, implemented in a two-phase procedure with a vast computing cost. First, a number of models of a same architecture but in different scales are enumerated, with the unknown parameters estimated under the maximum likelihood (ML) principle. Second, one of typical learning theories, being different from a ML principle, is applied to select the best model. Alternatively, we introduce a statistical theory that integrates parameter learning, model selection, and regularization from a unified perspective under the name of Bayesian Ying-Yang (BYY) learning. The parameter learning part of BYY learning consists of determining not only unknown parameters but also an inner code of each sample, while the model selection part is searching a BYY system with both a scale for each inner code and a scale for accommodating unknown parameters. Both parameter learning and model selection are made under a same learning principle called best Ying-Yang harmony, implemented by the EM-like adaptive algorithms. The separation of the inner coding scale from the other scales leads to three favorable features. First, we get a criterion that outperforms typical model selection criteria in a two-phase implementation. Second, on several typical learning tasks, model selection can be achieved automatically during parameter learning and save the computing cost significantly. Third, it provides a framework to integrate the roles of regularization and model selection, and mutually one can compensate certain weakness caused by the other. Experiments on several typical learning tasks are given to demonstrate these features.

On a Fractional Continuous Formulation for Independence Number of a Graph

BALABHASKAR BALASUNDARAM AND SERGIY BUTENKO

Department of Industrial and Systems Engineering, Texas A&M University College Station

In this paper we characterize the local maxima of a continuous global optimization formulation for finding the independence number of a graph. Classical Karush-Kuhn-Tucker (KKT) conditions and simple combinatorial arguments are found sufficient to deduce several interesting properties of the local and global maxima. These properties can be utilized in developing new approaches to the maximum independent set problem.

Quadratic Programming Techniques for Graph Partitioning

WILLIAM W. HAGER

Department of Mathematics, University of Florida

In a seminal paper (*An efficient heuristic procedure for partitioning graphs*, Bell System Technical Journal, **49** (1970), pp. 291–307), Kernighan and Lin propose a pair exchange algorithm for approximating the solution to min-cut graph partitioning problems. In their algorithm, a vertex from one set in the current partition is exchanged with a vertex in the other set when the sum of the weights of cut edges is reduced. This algorithm along with the related Fiducia/Mattheyses scheme are incorporated in state-of-the-art graph partitioning software such as METIS. A quadratic programming-based block exchange generalization of the Kernighan and Lin algorithm is introduced. Numerical experiments indicate that algorithm exploiting block exchanges can yield a significant improvement in partition quality over pairwise exchange algorithms.

Optimal Mixed-Integer Programming and Heuristic Methods for a Bilevel Stackelberg Product Introduction Game

J. COLE SMITH, CHURLZU LIM AND AYDIN ALPTEKINOGLU

Industrial & Systems Engineering, University of Florida

We consider a scenario in which two firms are determining which products to develop and introduce to the market. In this problem, there exists a finite set of products and market segments, and each market segment has a priority list of products that they will purchase if available. The firms play a Stackelberg game, in which the leader firm acts first to introduce a set of products, after which the follower firm responds with its own set of products. The goal of the leader is to maximize its profit, assuming that the follower will attempt to minimize the leader's profit. We formulate this problem as a multi-stage integer programming problem amenable to decomposition techniques. Using this formulation, we develop three variations of an exact mathematical programming method for this problem, along with a family of heuristic procedures for estimating the optimal follower solution.

An Application of the Approach of Moreau-Panagiotopoulos in Electronics

SAMIR ADLY

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In this talk, we will show that the approach of Moreau-Panagiotopoulos constitutes a powerful key that can be used to write a precise and rigorous mathematical model describing the dynamics of circuits involving devices like Zener diodes and diacs (DIode Alternating Current). Other devices like thyristors, varactors and silicon controlled rectifiers can be discussed in the same way. Using the recession analysis we give some necessary and sufficient conditions for the existence and the stability of a finite semi-coercive variational inequality with respect to data perturbation. The particular case of linear complementarity problems LCP will be discussed also. Some applications of the abstract results in mechanics and in electronic circuits involving devices like deal diode and practical diode are discussed.

Correspondences between Affine Differential Geometry and Information Geometry

MATSUZOE HIROSHI

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Affine differential geometry studies hypersurfaces immersed into an affine space, and information geometry studies geometrical structures of sets of probability distributions. A key fact is that duality of affine connections arises naturally, then these geometries have common geometric ideas. For example, the canonical divergence, which is an asymmetric distance-like function, is an essential tool in information geometry. In fact, the canonical divergence can be generalized by an affine support function in affine differential geometry. In this talk, a tutorial of correspondences between affine differential geometry and information geometry will be given.

Legendre-Fenchel Transform and Biorthogonal Coordinates of Differentiable Manifolds

JUN ZHANG

Department of Psychology, University of Michigan

On a differentiable manifold with Riemannian metric g , a pair of coordinate systems are called “biorthogonal” if the Jacobian governing their coordinate transform turns out to equal the metric tensor g evaluated on the coordinate bases. Examples of biorthogonal coordinates are the natural and expectation parameters, respectively, of exponential families in statistics, where the set of probability density functions form a differentiable manifold endowed with a Riemannian metric given by Fisher information. In general, biorthogonal coordinates are the dual variables associated with a pair of convex functions that are mutually conjugate under the Legendre-Fenchel transform. Geometrically, the existence of biorthogonal coordinates on a Riemannian manifold is equivalent to the existence of two torsion-free, flat affine connections that are dual with respect to the metric g . In such a case, a parametric family of affine connections can be constructed from that all admit with parallel volume forms and therefore generate equiaffine geometry. Biorthogonal coordinates are also used for dualistic expressions of divergence functions on the manifold, measuring the asymmetric “distance” between two points - Bregman divergence is the canonical divergence defined on a dually flat Riemannian manifold.

Complementarity Problems and Applications

MICHAEL C. FERRIS

Computer Sciences Department, University of Wisconsin

While optimizers are familiar with complementary slackness as the optimality conditions of linear and nonlinear programming, complementarity problems arise naturally in many practical applications from engineering and economics. Examples include applied general equilibrium modeling, traffic network design, structural engineering and finance. Several examples will be outlined, together with an overview of modeling and solution techniques.

Recent interest in optimization problems with complementarity constraints, or the more general class of MPEC's has rekindled the interest in nonlinear programming approaches to treat complementarity conditions. We outline basic ideas, highlight several computational schemes and explain their utility by application.

Second-Order Global Optimality Conditions for Optimization Problems

XIAOQI YANG

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Second-order optimality conditions are studied for the constrained optimization problem where the objective function and the constraints are compositions of convex functions and twice strictly differentiable functions. A second-order sufficient condition of a global minimizer is obtained by introducing a generalized representation condition. Second-order minimizer characterizations for a convex program and a linear fractional program are derived using the generalized representation condition.

Global Equilibrium Search Applied to the Unconstrained Binary Quadratic Optimization Problem and the Weighted MAX-SAT Problem

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We describe two heuristic approaches for solving the unconstrained binary quadratic optimization problem and the weighted **MAX-SAT** problem based on a global equilibrium search (GES) framework. We investigate performance of the proposed approaches and compare them with the best available solvers on well-known benchmarks instances. The reported computational results indicate a high efficiency of the developed algorithms.