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Robust Global Optimization

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A wide class of global optimization problems have the form

$$min\{f(x)|g(x) \leq 0\} \quad (1)$$

where $g(x) = \max_{i=1,\ldots,m} g_i(x)$, and $f, g_i : R^n \to R$. Since computing a feasible solution to (1) may be very hard, most solution methods for (1) compute, for given tolerances $\epsilon > 0, \eta > 0$, an (ϵ, η) - approximate optimal solution, i.e. a point $\bar{x}$ such that :

1. $\bar{x}$ is an ϵ-feasible solution, in the sense that $g(\bar{x}) \leq \epsilon$;
2. $f(\bar{x}) \leq f(x) + \epsilon$ for all ϵ-feasible solutions x.

However this concept of approximation may not be quite adequate, since such an (ϵ, η) - approximate optimal solution may correspond to an objective function value far from the true optimal value, while being infeasible. We introduce a concept of essential ϵ - optimal solution, which corresponds to a more appropriate approximation of the optimal global solution, while being stable under small perturbations of the constraints. A general robust solution approach is proposed which can be applied whenever $f, g_i \in \mathcal{C}$, where $\mathcal{C}$ is a family of functions with the following property:

1. If $f \in \mathcal{C}, \alpha \in R_+$ then $\alpha f \in \mathcal{C}$; If $g_1, g_2 \in \mathcal{C}$ then max $g_1(x), g_2(x) \in \mathcal{C}$;
2. Any set of the form $D = \{x \in R^n | h(x) \leq 0\}$ with int $D \neq \phi$; satisfies cl(int D) = cl(D).

This property ensures that the family $\mathcal{F}(\mathcal{C}) := \mathcal{C} - \mathcal{C}$ is a vecrtor lattice and if every function f, g_i involved in a problem (1) are functions of the family $F(\mathcal{C})$ then computing an essential ϵ - optimal solution reduces to solving a sequence of robust problems of the form $\min\{f(x)|g(x) \leq 0\}$ with $h \in \mathcal{C}, g \in \mathcal{F}(\mathcal{C})$. Examples of $\mathcal{C}$:

1. family of convex functions
2. family of increasing functions

Numerical examples are discussed to illustate the approach.