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Duality in Extended Linear and Nonlinear Programming

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In a view that is widely held, duality enters optimization through the introduction of Lagange multipliers, which are tied to the imposition of equality or inequality constraints. In other words, duality in the first instance is concerned with dualization of constraints. Then further, under certain assumptions of convexity, the multipliers can be characterized as solving a dual problem. Even without convexity, the constraint dualization leads to bounds on the optimal value in the primal problem.

This view, however, is unnecessarily narrow. In fact, the idea that an optimization problem should be modeled with exact constraints is too narrow. From a broader perspective, the ingredients to optimization modeling are typically some simple, structural constraints on the decision variables, expressing their nonnegativity and some definitional relationships, and then also a number of functions of these variables which all are of importance to keep high, or as the case may be, low.

Faced with this, people often choose one of the functions for the objective and place constraints on the others. But there are many other approaches to modeling, which may be better, depending on the circumstances. These can include combined penalty expressions, for instance.

The interesting fact is that duality can fully be developed in this "extended" setting with all the advantages that one would expect. Much greater flexibility in handling applications is thereby achieved. Once a problem is set up in the broader framework, there are easy tricks for solving it anyway by the software already available for problems in the traditional framework.