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Analyzing the effects of inventory cost setting rules in a disassembly and recovery environment

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In this study we consider a disassembly and recovery facility receiving end-of-life products and facing demand for a specific part that is disassembled from the product and then recovered. The disassembly and recovery operations can be either performed before hand, or upon customer arrival. In the latter case, a discount on the selling price is applied to compensate the customer for waiting for the completion of the disassembly and recovery operations. One of the difficulties faced in planning for such a system is the determination of the opportunity cost associated with carrying recovered parts inventory. The difficulty arises in seeking the value added to the part given the costs incurred for maintaining the product return, disassembly and recovery costs and revenue earned from the hulk, that is the remaining product after the disassembly of the part. The main objective of the study is to investigate the effect of different rules to determine this opportunity cost on the performance of the system. Six rules are considered in the study. The performance of the rules is assessed by a computational study under an approximate inventory control policy.

Keywords: Recovery; Disassembly; Inventory holding cost

1. Introduction

The majority of today's products are complex assemblies that are manufactured using pre-manufactured parts. A considerable fraction of these products and their parts are designed and manufactured for durability. Therefore, when such a complex product is discarded at the end of its useful life by the customer, it may be possible to recover several parts from it, and restore them into operating condition by cleaning, replacing and refabricating some components of the part. These recovered parts then can be reused as (replacement) service parts for the products that are still in use. As processing end-of-life products to recover reusable parts and/or recyclable materials is gaining popularity (including computers, cameras, imaging equipment, cellular phones as well as automobiles), optimizing the operations management practices for product and part recovery is becoming increasingly important (e.g., Fleischmann *et al.* (2003), Guide *et al.* (2003) and Spengler and Schröter (2003)).

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A detailed review of the operations management, production planning studies for product and part recovery can be found in a recent book by Dekker *et al*. (2004). In general, the trade-off between the inventory ordering, holding, and shortage costs is considered in analyzing these problems. Traditionally, the researchers have included the cost of capital tied up in the inventory together with inventory handling expenses (including material handling, insurance, and taxes, which are also referred to as the out-of-pocket expenses) in evaluating the cost of inventory. Typically, inventory holding cost per unit per unit time is charged as a fraction of the total value of the inventory to reflect the time value of the money tied up in stock. Then this inventory holding cost per unit per unit time is included in the model with an objective of minimizing cost or maximizing profit per unit time. This average cost (AC) approach is a well-accepted and widely used approximation, however it gives exact results only for a limited number of environments. Several authors, including Grubbström (1980), argue that exact discounted cast flow analysis, with the net present value of the cost or profit, should be carried out for exact analysis.

In general, setting inventory holding rates in the context of recovery systems is not straightforward. Under a particular class of inventory policies in a stochastic manufacturing and remanufacturing environment and using the NPV criterion, Teunter *et al*. (2000) investigate the performance of alternative rules to set the inventory holding cost rates for unremanufactured, remanufactured, and new items via simulation. In related work, Teunter and Van der Laan (2002) compare nearly optimal NPV and AC criterion policies in a deterministic manufacturing and remanufacturing environment with disposal option. By comparing the average annual profit function under the AC criterion with the annuity stream under the NPV criterion, they determine the values of the inventory holding cost rates such that these two systems yield the same manufacturing and remanufacturing batch sizes. However, the rates are not robust with respect to problem sets considered in approximating the NPV approach. Van der Laan (2003) considers the stochastic version of the same problem, and shows that, for moderate values of the return rate, the traditional way of setting holding cost rates in the AC and NPV approaches yields similar results when the remanufacturing set-up cost is negligible. When the remanufacturing set-up cost is significant, the 'right' holding cost rates are determined by studying the deterministic counterpart of the system under consideration. The results are very different from the holding cost rates that are set under the traditional way of setting the holding cost rates to unit (re)manufacturing cost times the interest rate. Teunter (2001) considers a disassembly system, and determines the inventory holding cost rates for the subassemblies from the NPV point of view for a given bill of materials structure.

The main focus of our study is to investigate the effect of inventory holding cost setting rules on the system performance for a disassembly and recovery system. Similar to the previous studies we consider alternative inventory holding cost setting approaches, determine the optimal operating policy parameters under these different approaches, and investigate the differences in system performance. Our study is different from those in the literature briefly reviewed above with respect to the following:

- The majority of the current work focuses on a joint manufacturing and remanufacturing environment, where there are perfectly substitutable new and remanufactured products in the system. The analysis is carried out under the assumption that new and remanufactured items have the same inventory value. In our work, we focus on a disassembly and recovery environment, where there are end-of-life products and parts that can be recovered from these products. Our analysis primarily focuses on how the inventory cost should be specified for recovered parts.

- We consider an average profit model in a stochastic environment, for which we do not have closed form solutions due to the complexity of the system and the proposed inventory policy. Moreover, NPV analysis for this system is quite difficult. Therefore, we are not able to analytically derive the holding cost rates for the average profit model that yields the same profit value as in the NPV analysis (as in Van der Laan (2003)). Rather than carrying out a simulation study to assess the differences of the proposed holding cost setting rules, we investigate various performance measures that capture the behaviour of different rules via analytical modeling.

The remainder of the paper is organized as follows. In section 2 we describe the system considered, our modeling assumptions, the inventory control policy considered, and the holding cost setting rules proposed in detail. In section 3 the results of the computational study are presented. Finally, in section 4 we give some concluding remarks.

2. Model development

In this section, we describe the system that we focus on in our study and discuss our modeling assumptions. We then present the inventory holding cost setting rules and inventory control policy we consider. We conclude this section by discussing how we calculate the optimal operating parameters for the inventory control policy.

2.1 *System description*

Our work is primarily motivated by a vehicle salvaging facility. The facility receives end-of-life vehicles from final owners. These are disassembled, and particular (high-value) parts, e.g., engine or transmission, are recovered. In the remainder of the description of the system, we will assume that the salvaging facility concentrates on the recovery of the engines. Upon disassembly of the engine, the remainder of the end-of-life vehicle, i.e., also referred to as the *hulk*, is sold to a bulk metal recycling facility. The recovered engines are stored to satisfy the demand for used engines in the automotive parts market. The engines that are recovered by salvaging facilities are typically purchased by independent automotive repair shops, or individuals who repair their vehicles themselves, i.e., do-it-yourself customers. In the automotive service parts industry, a customer who purchases a replacement service part is required to return the defective part. This is not the case for the system we consider, as the salvaging facility accepts end-of-life vehicles for processing only, and there is no need for a buyer to return a used engine to the facility. Hence, the final owner market of the end-of-life vehicles and the buyer market for the recovered engines are independent and non-overlapping.

Usually, the demand for recovered engines and the returns of end-of-life vehicles are not high in volume. Moreover, the make and model of the end-of-life vehicles returned to the facility may have a large variety. As a result, the disassembly and recovery operations are not standardized, and, hence, are not carried out in a continuous fixed-volume basis. Moreover, the major production resource is labor, and, therefore, the capacity adjustments are relatively easy in the case of peak demand periods. As a result, the lead times for the disassembly and recovery operations are relatively short and the customers wait for these operations to be carried out when there is an end-of-life product that can be disassembled. However, in that case, a price discount can be offered to the customer to compensate for the inconvenience.

The majority of these facilities store both end-of-life vehicles from which the engines can be disassembled for recovery as well as the recovered engines themselves. The stock for the end-of-life vehicles serves as a secondary source to satisfy the demand for the engines. That is, if the demand for recovered engines is more than the number of engines in the inventory and there are end-of-life vehicles in the system, then these vehicles are disassembled and the engines are recovered. Moreover, a closer examination of the salvaging facility practices reveals that the stock of end-of-life vehicles creates an additional revenue stream for the system as well. If the engine of an end-of-life vehicle can be reused, it is highly likely that some other parts on the vehicle also have some recoverable value (such as facia components, distributor, alternator). Typically, an individual customer or a professional repairer, who needs a replacement part for which supply is very limited may visit a salvage facility to examine the vehicles on site. If an end-of-life vehicle in the facility has the part that the customer needs, then the customer removes the part from the vehicle and the salvaging facility collects some revenue from this transaction. We provide a simple sketch of the system in figure 1.

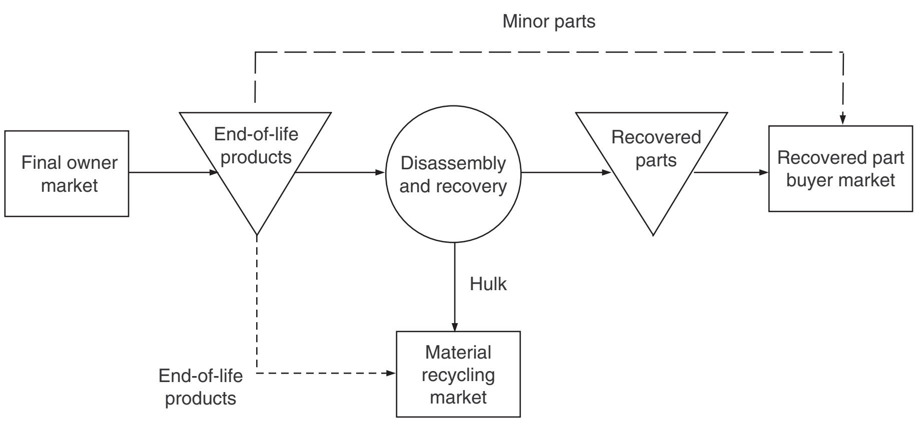

Figure 1. A simple sketch of the system.

2.2 *Problem statement*

Recovery systems that are similar to the vehicle salvaging facility we describe above have two main stocking points: end-of-life product and recovered part (which we refer to as the major part) inventories. Recovered parts in stock are used to satisfy external demand for the major part, whereas end-of-life products are kept as a buffer to harvest additional major parts as demand arises as well as to satisfy the demand for some other parts. In such a disassembly and recovery environment the main inventory decisions involve (i) whether or not to accept an end-of-life product return to the system, and (ii) when and how many end-of-life products to disassemble to recover the major part. The main trade-off in keeping product or part inventory is the relative revenue obtained from, and the cost associated with, satisfying major part demand from either of these two stock points. As in all inventory systems, estimating the revenue and cost parameters is crucial in making these decisions correctly. In our system, the revenues collected for satisfying the demand for the major part from the part or the product inventory are non-identical due to the price discount applied. Similarly, the inventory values of recovered parts and end-of-life products are non-identical due to the different physical characteristics, i.e., the volume and space they occupy, as well as the opportunity cost associated with keeping them in the system.

In the operations management literature, it is well accepted that the opportunity cost of carrying inventory is ‘some’ function of the total cost added for the item. However, it is not straightforward to determine the opportunity cost of holding inventories for recovered parts in a disassembly and recovery environment. It is not an easy task to determine the total value added to a recovered part, as it is not clear how to account for the costs incurred for end-of-life product returns and the salvage value of the hulk. The main objective of our study, therefore, is to assess the effect of different approaches to specifying the opportunity cost of holding recovered parts inventory on the performance of the system.

2.3 *Modeling assumptions*

In our analysis, we assume that the end-of-life products arrive at the salvage facility according to a Poisson process with mean rate λ_p and the facility incurs a unit cost of c_p for each end-of-life product that is received. This cost includes all costs associated with receiving one unit of end-of-life product return, including the premium that may have to be paid to the final owner and/or the associated transportation expenses. A unit disassembly cost c_d is incurred to disassemble the product. After the disassembly operation is performed and the part is reclaimed, the remainder of the product is sold to collect a unit salvage revenue of p_h which is associated with the recyclable value that resides in the hulk. The part has some recyclable material value also, and, hence, is associated with a unit salvage revenue of p_s. A unit recovery cost of c_r is incurred to recover a part, including all the costs associated with preparing the part for selling to a customer. If both the (end-of-life) product and the (recovered) part stocks are full, the salvage facility sells the end-of-life product directly for material recovery (without disassembling the product to recover the part), and a total revenue of $p_h + p_s$ is obtained. Finally, H_p is the unit inventory holding cost per product per unit time.

The demand for the part is also driven by a Poisson process with a mean rate of λ_c, and it can be satisfied by either the product or part inventory. If the demand is satisfied from the part inventory, a unit revenue of p_c is obtained. If the demand is satisfied from the product inventory, disassembly and recovery operations are still required, and, due to the inconvenience to the customer for having to wait for the completion of these operations, a unit revenue of p_p, where $p_p \leq p_c$, is collected. If the demand for the part cannot be satisfied, i.e., both product and part stocks are empty when the demand arrives, unsatisfied demand is lost, incurring a unit loss of goodwill cost of b_c. H_c is the unit inventory holding cost per part per unit time.

Since there are a large number of other parts that can be removed from the product for sale but the demand is relatively low, it is hard to keep track of the availability of these parts individually. To consider the effect of these parts, therefore, we consider a 'fictitious' part, which we refer to as the *minor* part in the remainder of our discussion. We assume that the demand for the minor part also arrives according to a Poisson process with a mean rate of λ_m, and the respective revenue of p_m is earned as long as there is a product in the inventory. The disassembly, recovery, and lost sale costs are ignored for the minor parts. We also assume that the minor part can be removed without damaging the major part. The notation used is summarized in table 1.

2.4 *Specification of the holding costs*

Recall that H_p and H_c are the unit inventory holding cost of products and parts, respectively. Unit inventory holding cost is comprised of two components:

(i) *out-of-pocket cost* that is related to the physical storage and handling of the products/parts; and

Table 1. Notation.

λ_p	Mean return rate of the end-of-life product per unit time
λ_c	Mean demand rate of the major part per unit time
λ_m	Mean demand rate of the (fictitious) minor part per unit time
p_c	Unit revenue earned when major part demand is satisfied from part inventory
p_p	Unit revenue earned when major part demand is satisfied from product inventory
p_m	Unit price of the (fictitious) minor part
p_h	Unit salvage value of the hulk
p_s	Unit salvage value of the major part
c_p	Unit cost of acquiring the end-of-life product
c_d	Unit cost of disassembly for the end-of-life product
c_r	Unit cost of recovering the major part
b_c	Unit lost sale cost for the major part
i	Inventory carrying charge per period
H_p	Unit total holding cost for the product per unit time
H_c	Unit total holding cost for the major part per unit time
h_p	Unit out-of-pocket holding cost for the product per unit time
h_c	Unit out-of-pocket holding cost for the major part per unit time

(ii) *opportunity cost* (cost of capital) that is proportional to the amount of capital tied up in inventory, which is a function of the value of the product/part and inventory carrying charge i used by the facility.

Let h_p and h_c denote the out-of-pocket inventory holding cost for the products and parts, respectively. For a product, we can set the total unit holding cost to $H_p = h_p + ic_p$ under the traditional way of setting inventory holding cost rates, where c_p is the initial acquisition cost of the end-of-life product. For a major part, however, specifying the unit holding cost is not straightforward, due to the difficulty associated with specifying the cost of value added to the part, since the acquisition cost paid for the end-of-life product and the cost of disassembly are incurred *both* for the major part and the hulk.

We consider three approaches to determine the inventory value of the major part:

(i) physical measure based;
(ii) market value based; and
(iii) recovered value approaches.

Under the first two approaches the hulk is treated as a product of the system. The joint cost of $c_p + c_d$ is split between the major part and hulk according to a certain criterion with a fraction of f to the major part. That is, the unit inventory holding cost for a major part is given by $H_c = h_c + i[(c_p + c_d)f + c_r]$. Under the physical measure based approach, the allocation criterion is based on the physical characteristics of the major part such as its weight, volume, or the number of components that a product contains. Under the market value based approach, the allocation depends on the market value of the part, i.e., the revenue that can be collected for the part. Under the recovered value approach, the hulk is treated as a by-product of the system, and the revenue earned by selling the hulk may or may not be considered to be a reduction on the total inventory value of the part. The following inventory holding cost setting rules are included in our study:

A. Physical measure based approach
- A.1 *Volume or weight of the components*. In some cases, the out-of-pocket inventory holding costs may be proportional to the relative volume or weight of the components. We may assume that the components of a product share the opportunity cost of holding inventory similarly, i.e., according to their relative volume or weight. In this case, the allocation fraction for the opportunity cost can be based upon the ratio of the out-of-pocket inventory holding costs incurred for the product and the major part. That is, we can set $f = h_c/(h_p + h_c)$.
- A.2 *Number of components*. As a product generates one unit of the major part and one unit of the hulk, we can assume that these two components of the product are 'uniform', and that the cost can be allocated equally between them. That is, we can set $f = 1/2$.

B. Market based approach
- B.1 *Sales value approach*. It may be possible to set the inventory carrying cost proportional to the sales price of a component of the product. In this case the allocation fraction for each component is proportional to its sales price. That is, the sales price for the hulk is p_h and the sales price for the major part is p_c, and we have $f = p_c/(p_c + p_h)$.

B.2 *Estimated net realizable value*. It may also be possible to set the inventory carrying cost proportional to the net revenue obtained for each component. In this case, the allocation fraction for each unit is proportional to its net revenue. That is, the net revenue collected for the major part is $p_c - c_r$, and the net revenue collected for each hulk is p_h, and we have $f = (p_c - c_r)/(p_c - c_r + p_h)$.

C. Recovered value approach

C.1 *Recovered hulk value*. Since the hulk is a by-product and is associated with some revenue, the revenue from material recovery pays some of the costs incurred to disassemble a product and recover the major part. If we take this into account, then the inventory value of a major part can be considered to be equal to $\max\{c_p + c_d + c_r - p_h, 0\}$, yielding a unit inventory holding cost of $H_c = h_c + i[\max\{c_p + c_d + c_r - p_h, 0\}]$.

C.2 *No recovered value*. Since the disassembly of the product to recover the major part is the core business of the salvaging facility, the hulk is a by-product of this process. Hence, we may ignore the revenue collected from hulk sales and allocate all of the acquisition, disassembly, and recovery costs to the major part. In othe words, the inventory value of the major part is equal to $c_p + c_d + c_r$, yielding a unit inventory holding cost of $H_c = h_c + i[c_p + c_d + c_r]$.

2.5 *Inventory control policy*

The optimal inventory control policy structure for such a system can be determined by utilizing the continuous time Markov Decision Processes approach. However, such an approach requires the analysis of two-dimensional state space with a large action space. Moreover, even if such an optimal policy is characterized, it may be too complicated for implementation in practice. Therefore, we restrict ourselves to a class of suboptimal policies, which we denote by (S_p, s_p, S_c, s_c), where

- S_p is the maximum level of product inventory allowed,
- s_p is the reservation (safety stock) level for the product inventory, i.e., the minimum level of product inventory required for the disassembly of a product that is already in inventory upon demand occurrence,
- S_c is the maximum level of part inventory allowed, and
- s_c is the reservation (safety stock) level for the major part inventory, i.e., the maximum level of major part inventory required for the disassembly of a product that is already in inventory upon demand occurrence.

The policy is developed in such a way that immediate disassembly of an end-of-life product return and recovery of the major part is prioritized as long as the maximum level of part inventory, S_c, is not exceeded. When the inventory level for the major parts is equal to the maximum allowable level of major parts, S_c, the returned products are kept in the product inventory as long as the product inventory is not above the respective maximum allowable level, S_p. If at the time of an end-of-life product arrival, there are S_c and S_p items in the major part and product inventory, respectively, then the returned product is sold immediately for material recovery. In order to keep discounted part sales at a reasonable level, we impose a certain reservation (safety stock) level s_c under which already available products are disassembled and recovered upon demand occurrence. Note that the product inventory does not only serve as a secondary source to satisfy major part demand, but it also is utilized to satisfy the demand for the minor parts. Therefore, we also put an additional restriction for triggering the disassembly and recovery operation upon demand occurrence; the product inventory should include at least s_p items. When a demand for the major part arrives, if the major part inventory is empty, but there are some products in the product inventory, then a product is disassembled and the major part is recovered to satisfy the demand. If the product inventory is also empty, then the demand for the major part is lost. When an end-of-life product return arrives, if both the product and part inventories are full, then the product is salvaged directly.

We note that the arrival of demand for the minor part does not change the state of the system, since we assume that there are infinitely many of them on the product and removal of a minor part does not damage the major part. Therefore, we do not track the inventory level for the minor parts. When the demand for the minor part arrives, if there is a product in the product inventory, then the demand for the minor part is satisfied. Otherwise, the demand cannot be satisfied, and is lost.

Let I_p and I_c denote the inventory level of the product and major part, respectively. Under the policy we consider, there are two events that change the state of the system, which are the arrival of an end-of-life product or a demand for the major part. These changes and the required conditions are as follows.

(1) When an end-of-life product is received,
- If $I_c < S_c$, then the product is disassembled. The major part is recovered, and is stored in the part inventory, i.e., I_c is increased by one. The hulk is sold at p_h for material recovery.
- If $I_c = S_c$,
  - if $I_p < S_p$, then the product is kept in the product inventory, i.e., I_p is increased by one.
  - if $I_p = S_p$, then the product is sold directly for material recovery for $p_h + p_s$.

(2) When a demand for the major part occurs,
- If $I_c > 0$, the demand is satisfied from part inventory earning a unit revenue of p_c, i.e., I_c is reduced by one.
  - if $I_c \leq s_c$ and $I_p \geq s_p$, a product is disassembled and the part is recovered to be stored in the part inventory, i.e., I_p is reduced by one and I_c is increased by one. A unit revenue of p_h is earned by selling the hulk for material recovery.
- If $I_c = 0$,
  - if $I_p > 0$, the demand is satisfied from the product inventory earning a unit revenue of p_p, i.e., I_p is reduced by one.
  - if $I_p = 0$, the demand is lost incurring a unit loss of goodwill cost of b_c.

We note that the reservation stock level of the product inventory is ignored upon a product return (when new product supply becomes available). That is, if the major part inventory is below its maximum allowed level, the product return is used to replenish the major part inventory *regardless* of the level of the product inventory. However, the reservation stock level of the product inventory is taken into account upon a demand occurrence (when available product supply needs to be adjusted). That is, if the major part inventory drops below its reservation level, the existing product inventory is used to replenish the major part inventory *only if* the product inventory is above its reservation level.

Although this may seem to be an inconsistent replenishment strategy, it is mainly due to the randomness in product supply and the fact that the facility has no control of the supply process. If the product inventory status is taken into account upon both events (a product return and a demand occurrence), the policy would in fact be prioritizing the replenishment of the product inventory, which is a secondary source to satisfy the major part demand. Moreover, if replenishing the major part inventory is prioritized upon both events, the opportunity to satisfy the minor part demand would be missed, and the flexibility of satisfying the major part inventory from product inventory at a reduced revenue would not be exploited.

2.6 *Derivation of the profit function and steady-state probabilities*

Since both arrival of end-of-life products and occurrence of major part demand are assumed to be Poisson processes, for given (S_p, s_p, S_c, s_c), the system can be modeled as a two-dimensional Markov process on the state space

$$\begin{aligned}\mathcal{M} = \{(I_p, I_c) \mid (I_p \in \{0, 1, \ldots, s_p - 1\}, I_c \in \{0, \ldots, S_c\})\\ \cup (I_p \in \{s_p, \ldots, S_p\}, I_c \in \{s_c + 1, \ldots, S_c\})\}.\end{aligned}$$

Let $\pi_{j,k}$ be the steady-state probability that the inventory levels for the product and major part are j and k, respectively. The steady-state balance equations of the underlying process can be stated as follows:

$$\begin{aligned}
(\lambda_c + \lambda_p)\pi_{j,k} &= \lambda_c \pi_{j,k+1} + \lambda_p \pi_{j,k-1}, && 0 \le j \le s_p - 1, k < S_c,\\
& && j \ge s_p, s_c + 2 \le k < S_c,\\
(\lambda_c + \lambda_p)\pi_{j,S_c} &= \lambda_p \pi_{j-1,S_c}, && 0 \le j < S_p,\\
(\lambda_c + \lambda_p)\pi_{j,s_c+1} &= \lambda_c \pi_{j+1,s_c+1} + \lambda_p \pi_{j,s_c}, && s_p \le j < S_p,\\
(\lambda_c + \lambda_p)\pi_{S_p,k} &= \lambda_c \pi_{S_p,k+1}, && k < S_c,\\
\lambda_c \pi_{S_p,S_c} &= \lambda_p(\pi_{S_p-1,S_c} + \pi_{S_p,S_c-1}).
\end{aligned}$$

As explained previously, priority is given to satisfy the demand for the major part from the part inventory. If the major part inventory runs out of stock, a product in the product inventory is disassembled, if available, and the part is recovered to satisfy the demand. Therefore, the average net revenue from part sales can be expressed as follows:

$$\begin{aligned}R_c(S_p, s_p, S_c, s_c) = \lambda_c\Bigg((p_c - c_d - c_r + p_h)\Bigg(1 - \sum_j \pi_{j,0}\Bigg)\\ + (p_p - c_d - c_r + p_h)\sum_{j \ne 0} \pi_{j,0}\Bigg). \qquad (1)\end{aligned}$$

Note that in (1), $1 - \sum_j \pi_{j,0}$ and $\sum_{j \neq 0} \pi_{j,0}$ give the long-run fraction of the time that the demand for the major part is directly satisfied from the part inventory and product inventory, respectively. With these fractions, the revenue of either p_c or p_p is earned depending on the source utilized to satisfy the demand with rate demand rate λ_c. Equation (1) captures the *net* revenue from major part sales, hence it includes the unit disassembly and recovery costs, as well as the revenue earned from sale of the hulk for material recovery.

The demand for the major part is lost when both stock points are empty at the demand rate for the major part, λ_c. Then, the long-run average penalty cost for lost sales is given by

$$C_b(S_p, s_p, S_c, s_c) = \lambda_c b_c \pi_{0,0}. \tag{2}$$

Our model assumes that the demand for the minor parts are satisfied as long as there is product in the product inventory with the corresponding demand rate of λ_m. Since we assume that the facility does not incur any cost for the disassembly of these parts, the average net revenue earned by the minor part sales is given by

$$R_m(S_p, s_p, S_c, s_c) = \lambda_m p_m \sum_{j \neq 0} \sum_k \pi_{j,k}. \tag{3}$$

According to our control policy, the end-of-life product returns are rejected (that is, they are sold for material recovery directly) when $I_p = S_p$ and $I_c = S_c$. Therefore, in the long run, as the fraction of time of π_{S_p, S_c}, salvage revenue of $p_s + p_h$ is earned with return rate λ_p. Therefore, the salvage revenue can be expressed as follows:

$$R_s(S_p, s_p, S_c, s_c) = \lambda_p (p_h + p_s) \pi_{S_p, S_c}. \tag{4}$$

The long-run inventory holding costs can be expressed as

$$C_h(S_p, s_p, S_c, s_c) = \sum_j \sum_k (jH_p + kH_c) \pi_{j,k}. \tag{5}$$

Finally, the long-run average cost of receiving end-of-life product returns is independent of the control policy employed and can be expressed as

$$C_p = \lambda_p c_p. \tag{6}$$

The problem of finding the optimal policy parameters (S_p, s_p, S_c, s_c) maximizing the long-run average profit function can be stated as follows:

$$\begin{aligned} \max \Pi(S_p, s_p, S_c, s_c) = {} & R_c(S_p, s_p, S_c, s_c) - C_b(S_p, s_p, S_c, s_c) \\ & + R_m(S_p, s_p, S_c, s_c) + R_s(S_p, s_p, S_c, s_c) \\ & - C_h(S_p, s_p, S_c, s_c) - C_p, \end{aligned} \tag{7}$$

subject to

$$s_c \leq S_c, \tag{8}$$

$$s_p \leq S_p, \tag{9}$$

$$S_p, s_p, S_c, s_c \in \{0, 1, \ldots\}. \tag{10}$$

$R_c(S_p, s_p, S_c, s_c)$, $R_m(S_p, s_p, S_c, s_c)$, $R_s(S_p, s_p, S_c, s_c)$, $C_b(S_p, s_p, S_c, s_c)$, $C_h(S_p, s_p, S_c, s_c)$ and C_p are explicitly stated in (1), (2), (3), (4), (5), and (6), respectively. Constraints (8) and (9) are required to guarantee that the reservation levels at the stock points do not exceed the maximum inventory levels permitted. Constraint (10) ensures that the policy parameters take integral values.

3. Computational study

We conduct an extensive computational study to investigate the effect of inventory holding cost setting rules on the performance of the system. To assess the performance of the system, we consider the profitability of the recovery business, the service levels for the major and minor parts, the overall service performance, the maximum inventory level in the system, and the total out-of-pocket inventory holding cost incurred.

In our experiments, we fix the following parameters: unit revenue of satisfying the major part demand from the part inventory (p_c), unit revenue of salvaging the hulk (p_h), unit cost of not satisfying the major part demand (b_c), unit out-of-pocket holding cost for the product (h_p), unit cost of receiving a product return (c_p), inventory carrying charge (i), average end-of-life product return rate (λ_p), and average demand rate for the minor parts (λ_m). Their values are set as follows: $p_c = 300$, $p_h = 40$, $b_c = 0$, $h_p = 10$, $c_p = 200$, $i = 0.02$, $\lambda_p = 10$, and $\lambda_m = 1$.

We consider two different scenarios for the unit disassembly (c_d) and recovery (c_r) costs. For the recovery cost dominant case, we use $c_d = 25$ and $c_r = 50$. For the disassembly cost dominant case, we use $c_d = 50$ and $c_r = 25$. The relative values of disassembly and recovery costs depend on the specific characteristics of the product and the system. As the labour cost is the primary cost component for both operations, the magnitude of these cost elements may vary depending on the complexity of the product structure, operation times, and required labour skills. By setting the unit recovery and disassembly costs as above, we ensure that the profit margin of the major part is the same under both cases, allowing us to concentrate on cost adding structures of the operations independent of the unit profit margin.

In our experiments, we restrict our attention to the cases where salvage values and out-of-pocket holding costs are proportional, i.e., the cases where $p_s/p_h = h_c/h_p = w$ holds, where w represents the contribution of the major part to the product's weight. We consider three different levels for the parameter w such that

(i) $w = 1/4$ ($p_s = 10$, $h_c = 2.5$);
(ii) $w = 1/2$ ($p_s = 20$, $h_c = 5$); and
(iii) $w = 3/4$ ($p_s = 30$, $h_c = 7.5$).

The unit revenue earned by satisfying a major part demand by disassembly and recovery upon demand occurrence, p_p, is defined as $p_p = p_c * (1 - d)$, where d is the ratio of the discount given. We consider six levels for the discount factor, $d = 0.00$, 0.01, 0.05, 0.10 and 0.20. The significance of the minor component demand is investigated by considering two different price levels $p_m = 50$ and 100. Three different levels of the average demand rate for the major part are considered, $\lambda_c = 0.8$, 0.9 and 1.1. The levels of the experimental factors are summarized in table 2.

Table 2. Experimental design.

Factor	Levels
$w = p_h/p_s = h_p/h_c$	1/4, 1/2, 3/4
$d = (p_c - p_p)/p_c$	0, 0.01, 0.05, 0.1, 0.2
p_m	50, 100
λ_c	0.8, 0.9, 1.1
(c_r, c_d)	(25, 50), (50, 25)

Each combination of the factors defines a problem instance. For each problem instance, we find the optimal inventory policy parameters, (S_p, s_p, S_c, s_c), through a complete search procedure for each of the six inventory holding cost setting rules. Note that since we do not have the closed form expressions for the steady-state probabilities, and, consequently, for the objective function, we do not have any concavity results. However, we expect the decision variables to take finite values due to the inventory holding costs involved in the model. Our procedure considers a search space defined by $S_c \in \{0, 1, \dots, 20\}$, $S_p \in \{0, 1, \dots, 20\}$, with all possible values of s_c and s_p. For each (S_p, s_p, S_c, s_c) quadruple the steady-state probabilities are determined using the balance equations, and the respective expected profit value is calculated. Upon completion of the search, if one of the decision variables (S_c and/or S_p) is within five units of proximity of the borders of the search space in the current best solution, then the search space is enlarged by adding 10 more S_p and S_c values together with all feasible s_c and s_p values. The raw results are available upon request from the authors. We summarize our observations on the performance measures we mention above.

3.1 *Profitability of part recovery business*

The set of problem instances we consider includes instances where the best alternative is to salvage the end-of-life product returns directly. That is, the optimal inventory control parameters are $S_p = s_p = S_c = s_c = 0$, suggesting that all product returns are salvaged for material recovery. Note that, in our experimental setting, we restrict ourselves to the instances where pure material recovery is not profitable, since the unit cost of receiving a product return is lower than its total salvage value, i.e., $c_p < p_h + p_c$. Therefore, for the problem instances where the pure salvaging policy is the best, the system makes negative profit.

Pure salvaging is the best policy, where (i) the demand for the major part is low; (ii) the salvage value and the out-of-pocket holding cost for the major part are high; (iii) the selling price of the minor part is low; and (iv) the unit recovery cost for the major part is high. For the disassembly cost dominant case, the out-of-pocket inventory cost together with the salvage value of the major part must be high, i.e., the parameter w should be high, in order for pure salvaging to be the best alternative. In particular, pure salvaging is the best alternative under all inventory holding cost setting rules and all discount levels for the recovery cost dominant case with parameters

(i) $\lambda_c = 9$, $p_m = 100$, $w = 3/4$;
(ii) $\lambda_c = 8$, $p_m = 100$, all w levels considered;

(iii) $\lambda_c = 9$, $p_m = 50$, $w = 3/4$; and
(iv) $\lambda_c = 8$, $p_m = 50$, all w levels considered as well as for the disassembly cost dominant case with parameters $\lambda_c = 8$, $p_m = 50$, and $w = 3/4$.

Among all the problem instances considered, there are several instances where the profitability of performing part recovery depends on the inventory holding cost setting rule. Under the recovery cost dominant case with parameters $\lambda_c = 9$, $p_m = 50$, $w = 1/2$ and all discount levels considered, the pure salvaging policy turned out to be the best under rules B.1, B.2 and C.2 (those that assign the highest value to the major part), whereas under the other rules some part recovery is performed. In general, we expect that assigning a high value to the major part makes the recovery option less attractive. The fact that our decision variables take only integer values makes this effect substantial.

3.2 *Customer service for the major part*

Under our model assumptions, the demand for the major part is satisfied as long as there is either a recovered part or an end-of-life product in the system. Hence, the probability that demand is satisfied for the major part is given by $\alpha_c = 1 - \sum_j \pi_{j,0} + \sum_{j \neq 0} \pi_{j,0} = 1 - \pi_{0,0}$. In general, for the problem instances where the average demand rate for the major part is lower than the average product return rate, at least 90% of the major part demand is satisfied under all inventory holding cost setting rules. For the other cases, whenever the recovery business is performed, the service rate attained is at least 95%. The low service rate in the case of high demand is mainly due to the scarcity of supply. The sensitivity of the major part service rate is negligible with respect to

(i) the cost adding structure (whether the disassembly or recovery cost is dominant);
(ii) the minor part price; and
(iii) the discount made in the case of part inventory stock out.

For each problem instance, excluding those where the best alternative is to salvage all product returns, we calculate the range of α_c values among all six rules included in the study. The maximum range of obtained α_c values is 0.012. Therefore, we conclude that when part recovery is profitable, the inventory cost setting rule in effect does not affect the major part service performance.

Although the overall service is not very sensitive to the inventory cost setting rule in effect, the source to satisfy the major part demand depends on the rule selected. In general, as the inventory value of the recovered parts increases, the probability that the demand is satisfied from the part inventory decreases. The effect diminishes as the discount offered for satisfying the demand from the product inventory increases. Moreover, in the case of a high price for the minor parts and salvage value of the part the effect is less substantial.

We denote the probability of satisfying major part demand from the part and the product inventory by $\alpha_{c,c}$ and $\alpha_{c,p}$, respectively. Figures 2(a) and (b) depict the values of $\alpha_{c,c}$ and $\alpha_{c,p}$, respectively, for a typical example for all the cost setting rules under their respective optimal inventory control parameters. When no discount is offered, the inventory cost setting rules that assign a high value to the part

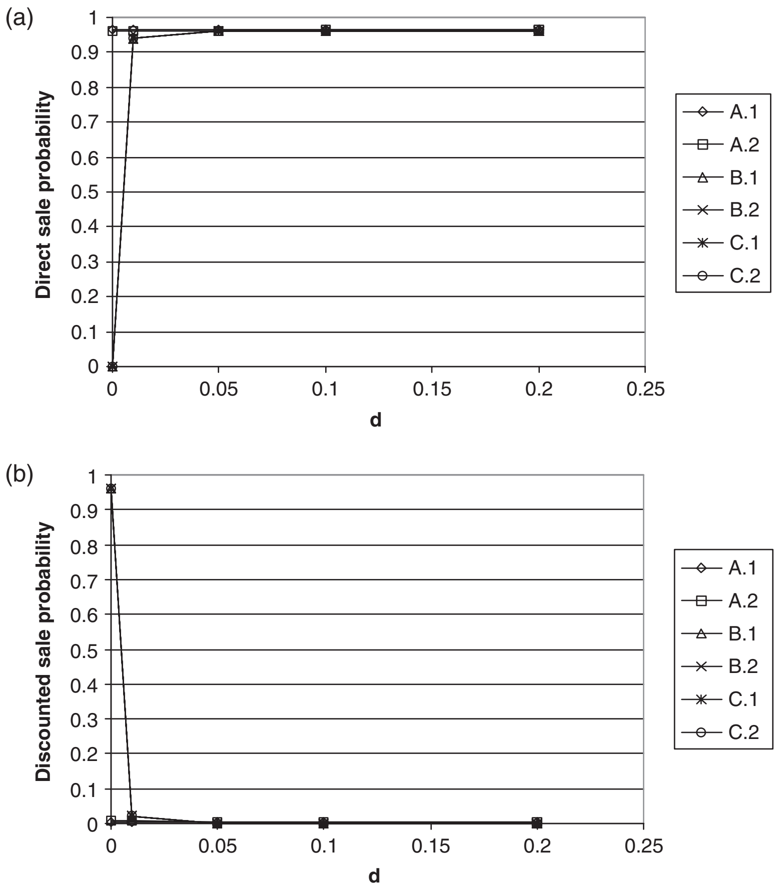

Figure 2. Probability of satisfying major part demand from (a) part inventory and (b) product inventory, i.e., $\alpha_{c,c}$ values, applying price discounts under different discount, d, values (for disassembly cost dominant case with $\lambda_c = 9$, $w = 3/4$, and $p_m = 100$).

inventory, namely B.1, B.2, C.1, and C.2 for this example, satisfy the major part demand entirely from the product inventory. Under A.1 and A.2, keeping the parts inventory is cheaper than keeping product inventory. Hence, when the discount rate is low, it is optimal to satisfy the entire major part demand from the product inventory. As the discount rate increases, thereby increasing the penalty incurred by satisfying the major part demand from the product inventory, the probability of satisfying the demand from the product inventory is very small under the cost setting rules considered.

Finally, in figure 3 the overall service for the major part, $\alpha_c = \alpha_{c,c} + \alpha_{c,p}$, is provided for the same example. It can be observed that the overall service is quite robust with respect to the cost setting rule in effect under all discount values considered. The rules that set the inventory values lower, i.e., A.1 and A.2, result in slightly better service. Note that under rule A.2 the increase in the service level when the discount rate increases is unexpected. We believe that this is due to the fact that the decision variables take only integer values.

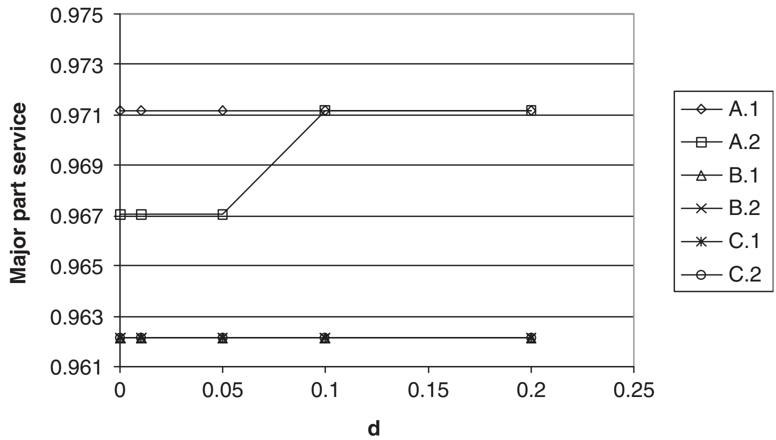

Figure 3. Overall probability of satisfying major part demand, i.e., α_c, under different discount, d, values (for disassembly cost dominant case with $\lambda_c = 9$, $w = 3/4$, and $p_m = 100$).

3.3 *Customer service for the minor part*

Although it is not the primary activity of the salvaging facility, satisfying the demand for minor parts may have importance. The probability of satisfying demand for minor parts is denoted by $\alpha_m = \sum_{j \neq 0} \sum_k \pi_{j,k}$ for our model.

Figure 4 shows the α_m values for the example in section 3.2. As can be observed, the rules assigning the highest value for the recovered part, B.1, B.2, C.1, and C.2 for this example, yield the highest level of service minor part demand. The difference in α_m values yielded under different rules is substantial for lower levels of the discount factor. Over all problem instances considered, the maximum difference recorded is 0.629. The deviation is usually high under lower levels of discount.

3.4 *Overall service performance*

The product and part inventory availability measure the performance of the system from the customer's perspective. To measure the system performance from the facility's perspective, we consider the following weighted service level measure that captures the proportion of the sale potential that is attained:

$$\bar{\alpha} = \frac{\lambda_c(p_c\alpha_{c,c} + p_c(1-d)\alpha_{c,p}) + \lambda_m\alpha_m}{\lambda_c p_c + \lambda_m p_m}.$$

We concentrate on a subset of the problem instances to investigate the performance of the inventory holding cost setting rules with respect to this measure. We consider four sets of problem instances. S.0 is the base scenario, where we have $\lambda_c = 9$, $p_m = 50$, and $w = 1/2$. We analyze the effect of the unit price of the minor parts in S.1, where we have $\lambda_c = 9$, $p_m = 100$, and $w = 1/2$. We study the contribution of the major part to the product in S.2, where we have $\lambda_c = 9$, $p_m = 50$, and $w = 1/4$. Finally, we examine the impact of the mean demand rate for the major part in S.3, where we have $\lambda_c = 8$, $p_m = 50$, and $w = 1/2$.

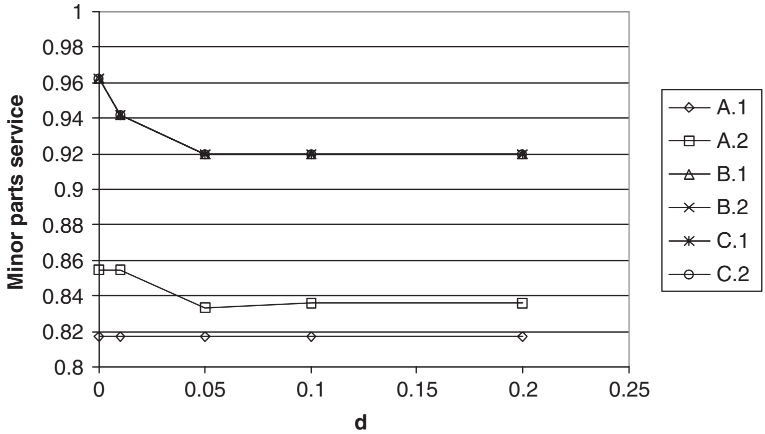

Figure 4. Probability of satisfying minor parts demand, i.e., α_p values, under different discount, *d*, values (for disassembly cost dominant case with $\lambda_c = 9$, $w = 3/4$, and $p_m = 100$).

Table 3. Percentage deviations in overall service level for recovery and disassembly cost dominant cases.

	d				
	0	0.01	0.05	0.1	0.20
Recovery cost dominant case					
S.0	0.26	0.26	0.26	0.24	0.25
S.1	0.82	0.82	0.83	0.84	0.86
S.2	1.13	1.13	1.13	1.14	1.15
S.3	0.48	0.35	0.32	0.33	0.38
Disassembly cost dominant case					
S.0	0.68	0.68	0.68	0.69	0.73
S.1	0.82	0.82	0.83	0.84	0.86
S.2	0.95	0.96	0.96	0.97	0.97
S.3	0.19	0.16	0.11	0.37	0.38

To investigate the deviation in the $\bar{\alpha}$ values under the different inventory cost setting rules considered, we consider the 'range' of the $\bar{\alpha}$ values, i.e., the difference between the maximum and minimum values obtained under all six rules, for each set of problem instances. To eliminate the variability caused by other problem parameters, we divide the 'range' by the 'mean' of the six $\bar{\alpha}$ values obtained, and multiply by 100 to represent the difference as a percentage. Table 3 summarizes the results for the selected four scenarios for recovery and disassembly cost dominant cases. As discussed earlier, for scenario S.0 under the recovery cost dominant case, some of the rules select pure salvaging as the best policy. For this set, we exclude these rules in calculating the range and average of the service levels observed. Therefore, the deviations are very small for this set of scenarios.

From the table it can be observed that deviations with respect to the overall service measure are very small. As the trade-offs become more substantial, i.e., as (i) the minor part price decreases; (ii) the contribution of the major part to both the volume and salvage value of the product decreases; and (iii) the demand for the major part increases, the deviations are larger than in the base case, i.e., we can identify differences between the rules. In general, over the entire set of problem instances considered, we do not observe large variations in overall service performance or the dominance of any specific rule.

3.5 *Maximum inventory levels allowed*

Although the rules do not exhibit any significant differences in service measures and revenue collected within the parameter set we consider, they differ in stocking decisions. Since the maximum inventory levels, i.e., S_c and S_p, bound the minimum reservation levels, i.e., s_c and s_p, we only examine the deviations in maximum inventory levels. Moreover, S_c and S_p are important since they define the storage space needed for product and part inventories.

In figures 5(a) and (b), optimal S_p and S_c values are depicted for the example we consider in section 3.2. It can be observed that, as the inventory value of the major part is set higher, the value of the optimal S_c decreases and S_p increases, as expected.

As the space requirements for the parts and products can be substantially different, we consider the range of $S_c/(S_c + S_p)$ values for each problem instance under all the inventory cost setting rules considered. Among all problems instances considered the largest range is 0.85.

3.6 *Out-of-pocket holding costs*

The total average out-of-pocket holding cost incurred under optimized policy parameters heavily depends on the inventory cost setting rule in effect. As in the case of the weighted service measure, we consider the ratio of 'range' to 'mean' average out-of-pocket holding cost multiplied by 100 to demonstrate the differences in the rules. Table 4 summarizes the results for the same four sets of scenarios considered in section 3.4.

As can be observed from the table, the effects of (i) the minor part price, and (ii) the contribution of the major part to the product are the same as in the case of the weighted service measure, i.e., decreases in these parameters differentiate inventory cost setting rules from each other. As opposed to the weighted service measure, decreasing the mean demand rate increases the differences in out-of-pocket holding costs incurred under different cost setting rules. In general, the percentage deviations defined in terms of the ratio of the range of values to their mean is much more substantial, particularly for scenario sets S.2 (with a lower contribution of the major part to product volume and salvage value) and S.3 (with a lower demand rate) under the recovery cost dominant case.

For scenario set S.2, the highest out-of-pocket cost figure is given by rule A.1, which allocates the product return and disassembly cost to the major part in proportion to its contribution to the product. In this scenario set, A.1 yields the lowest opportunity cost for carrying part inventory, and, as a result, it gives the

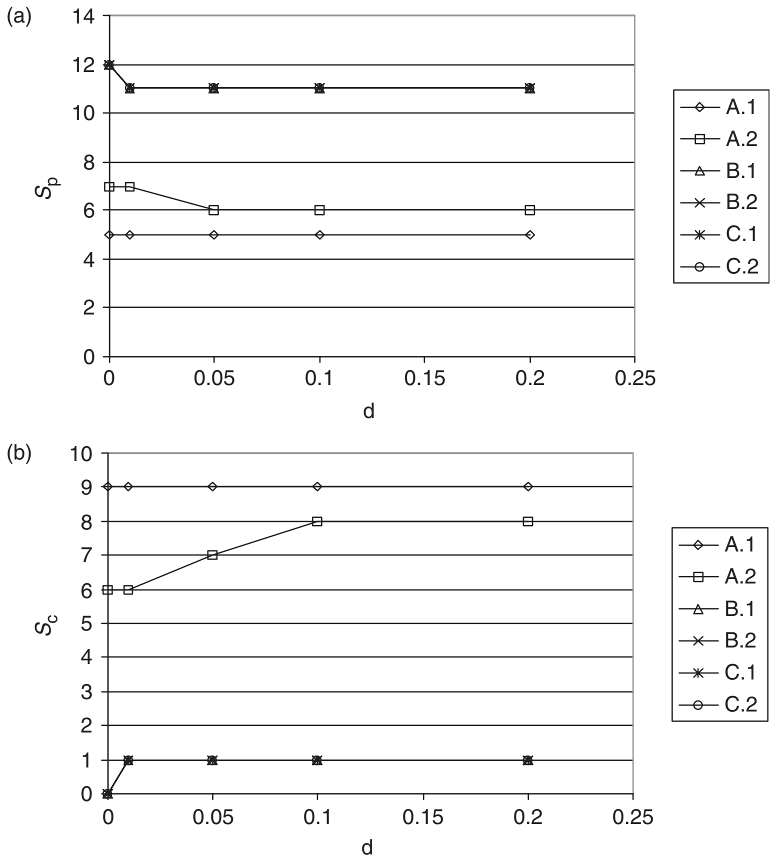

Figure 5. Maximum allowable (a) product and (b) part inventory levels under different discount, d, values (for disassembly cost dominant case with $\lambda_c = 9$, $w = 3/4$, and $p_m = 100$).

Table 4. Percentage deviations in out-of-pocket holding costs for recovery and disassembly cost dominant cases.

	d				
	0	0.01	0.05	0.1	0.2
Recovery cost dominant case					
S.0	1.52	1.52	1.52	0.30	0.03
S.1	2.54	10.99	11.07	11.03	10.98
S.2	30.14	30.14	30.14	29.99	31.10
S.3	96.36	87.60	90.70	85.47	92.19
Disassembly cost dominant case					
S.0	13.70	13.70	13.70	13.70	15.49
S.1	10.99	10.99	11.07	11.03	10.98
S.2	24.22	24.22	24.22	24.10	24.65
S.3	22.17	19.95	17.61	43.72	72.61

highest maximum part inventory level. The lowest opportunity cost is attained by either B.1, B.2, C.1, or C.2 over the range of discount factors considered. As a result, these rules allow high levels of part inventory and high inventory out-of-pocket costs, although the unit out-of-pocket holding cost is relatively low for the parts in scenario set S.2.

For scenario set S.3, the rules assigning a higher inventory value for the parts, namely B.1, B.2, C.1 and C.2, tend to carry more product inventory. Since the average demand rate is lower than the average return rate for scenario set S.3, the product inventory levels are higher. Since the unit out-of-pocket inventory cost is significantly higher for products than for parts, the effect is more substantial than for scenario set S.2. In this scenario set, A.1 and A.2, the rules that assign lower values to the parts adapt to increases in price discounts better; the decrease in optimal maximum product inventory levels is higher with increasing values of the discount factor under these rules. As a result, the difference is larger for higher levels of the discount factor.

4. Concluding remarks

Inventory holding cost rates are important parameters in the analysis of any production/inventory planning model. However, setting the opportunity cost of carrying inventories is not straightforward, particularly for disassembly and recovery environments.

In this study, we investigate the effect of different holding cost setting rules on the performance of a disassembly and recovery system where end-of-life products are received. The system faces demand for a certain part that can be recovered after the disassembly of the product. Our analysis reveals the following major insights under the inventory control policy considered and the parameter values included in the numerical study.

- Inventory holding cost setting rules can be important in determining the profitability of a recovery business for 'borderline' systems where the recovery option is barely attractive with respect to demand, revenue and cost schemes. In these cases, while determining whether or not to operate the recovery business, rather than using a result generated under a specific rule, considering all ranges of the holding cost values generated by different rules, and carrying out a what-if type of analysis under the net present value (NPV) criterion may generate more robust and correct results.
- The proportion of total possible revenue earned is quite robust with respect to the set value of the opportunity cost of carrying inventory. Similarly, the rules cannot be differentiated with respect to the percentage of major part customers satisfied. On the contrary, the percentage of minor parts customers satisfied heavily depends on the inventory holding cost set for the major part. None of the rules that we include in our study is observed to be dominating. In the cases where there is the possibility that the minor parts business becomes more important, hence satisfaction of minor part customers is important, a more detailed analysis should be applied.

- The rules considered yield significantly different maximum inventory levels allowed for the products and parts. In general, the higher the inventory holding cost for the parts, the smaller the maximum inventory allowed. The market based, i.e., B.1 and B.2, and recovered value approaches, i.e., C.1 and C.2, assign the highest inventory value for the parts. Depending on the other parameters of the system, this difference may lead to large differences in the average out-of-pocket inventory cost incurred. In real-life applications, usually the maximum inventory allowed is predetermined by other parameters, such as space availability. For most systems, as in the case of end-of-life vehicle disassembly, products and parts have completely different physical shapes and space requirements. In the case where the maximum available space is restricted for products and parts separately, the optimal policy parameters should be determined under this restriction. We believe that differentiation of the rules will be less significant in this case, since there will be a constraint on the decision variables, S_c and S_p.

There are a number of ways to extend the current study to cover other practical aspects. An immediate extension is to consider a multi-part system. In this case, the product structure (as studied by Teunter (2001)) and the disassembly sequence are important in determining the value added to each major part. Consideration of setup costs (due to transportation of the end-of-life products to the facility or the utilization of (semi-)automated disassembly and recovery operations) and/or yield loss and/or the probabilistic behaviour of customers (as not all customers may be willing to wait for the disassembly and recovery operations when the part inventory runs out of stock and there is at least a product in the inventory, at the given discount level) are other practically relevant and important issues that would complicate the analysis further. Finally, the investigation of other practical inventory control policies is also a potential extension of our work.

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